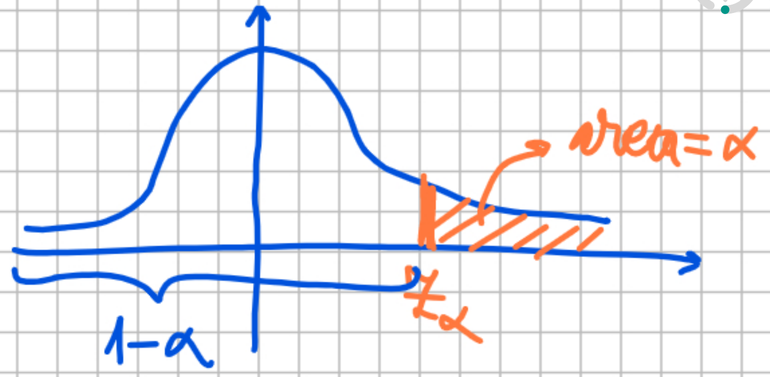
Lecture 6. Hypothesis testing.

Example 1. The number of concurrent users for some internet service provider has always averaged 5000 with a standard deviation of 800. After an equipment upgrade, the average number of users at 100 randomly selected moments of time is 5200. Does it indicate, at a 5% level of significance, that the mean number of concurrent users has increased? Assume that the standard deviation of the number of concurrent users has not changed.



$\bar{X} = 5200$

$\alpha = 0.05$

$H_0: \mu = 5000$ (μ_0)

$H_a: \mu \neq 5000$ ($>$ or $<$)

$H_0: \mu = 5000$

$H_a: \mu > 5000$ (one-tailed test)

Test statistic: $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{5200-5000}{\frac{800}{\sqrt{100}}} = \dfrac{200}{80} = \dfrac{20}{8} = \dfrac{5}{2} = 2.5$

$\mu_0 = 5000$

$\sigma = 800$

$n = 100$ (sample size)

$\bar{X} = 5200$ (sample mean)

$Z \sim N(0,1)$

$R = (z_\alpha, \infty)$ - rejection region

$z_\alpha = \text{qnorm}(1-\alpha, 0, 1) = 1.64$

$R = (1.64, \infty)$

$Z = 2.5 \in R = (1.64, \infty) \Rightarrow H_0$ is rejected

(we have enough evidence to reject H_0)

(we accept the claim that mr. of users has increased)