Lecture 4. Random vectors

Ex. 1. We distribute 3 balls a, b, c in 3 boxes.

X - nr. of non-empty boxes

$$X: \begin{pmatrix} 1 & 2 & 3 \\ \frac{3}{27} & \frac{18}{27} & \frac{6}{27} \end{pmatrix}$$

Y - nr. of balls in the first box

$$Y: \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{8}{27} & \frac{12}{27} & \frac{6}{27} & \frac{1}{27} \end{pmatrix}$$

a b c
c b

$\frac{3}{27}$

a b c
a c b
b a c
⋮
ab c
ac b
bc a

$\rightarrow \binom{3}{2} \cdot \binom{3}{1} \cdot 2$

ab c 3·2

a bc
a
bc

ab
c

Z = (X, Y) - represents the pairs (nr. of non-empty boxes, nr of balls in the first box)

$$Z: \begin{pmatrix} (1,0) & (1,1) & (1,2) & (1,3) & (2,0) & (2,1) & (2,2) & (2,3) & (3,0) & (3,1) & (3,2) & (3,3) \\ \frac{2}{27} & 0 & 0 & \frac{1}{27} & \frac{6}{27} & \frac{6}{27} & \frac{6}{27} & 0 & 0 & \frac{6}{27} & 0 & 0 \end{pmatrix}$$

$$P(Z=(i,j)) = P(X=i, Y=j) = P(X=i \mid Y=j)\cdot P(Y=j) \quad (X, Y \text{ are not indep.})$$
$$= P(Y=j \mid X=i)\, P(X=i)$$

$$P(Z=(1,0)) = P(X=1, Y=0) = P(X=1 \mid Y=0)\cdot P(Y=0) = \frac{2}{27}$$

$$Z : \begin{pmatrix} (1,0) & (1,3) & (2,0) & (2,1) & (2,2) & (3,1) \\ \frac{1}{27} & \frac{1}{27} & \frac{6}{27} & \frac{6}{27} & \frac{6}{27} & \frac{6}{27} \end{pmatrix}$$

Ch. 7

(67) a - freshmen
b - sophomores
c - juniors
d - seniors

X - freshmen + sophomores
Y - nr. of juniors
Z - nr. of seniors

a) sampling with replacement
n - sample size

PMF of (X,Y,Z): $P(X=n_1, Y=n_2, Z=n_3) = \frac{n!}{n_1! \cdot n_2! \cdot n_3!} \cdot p_1^{n_1} p_2^{n_2} p_3^{n_3}$

$n_1+n_2+n_3=n$

$(X,Y,Z) \sim Mult(n,p)$

$p = \left(\underbrace{\frac{a+b}{a+b+c+d}}_{p_1}, \underbrace{\frac{c}{a+b+c+d}}_{p_2}, \underbrace{\frac{d}{a+b+c+d}}_{p_3} \right)$

$\binom{n}{k}$