Test 1

- naive definition of probability and counting techniques
    - multiplication rule, permutations and combinations with/without replacement
    - inclusion-exclusion formula

- conditional probability, law of total probability, Bayes' rule

- discrete random variables and distrubutions
    - pmf and cdf
    - operations with random variables
    - expected value, variance
    - Bernoulli, binomial, geometric, negative binomial, hypergeometric and Poisson distributions

- continuous random variables
    - pdf, cdf - definition and properties
    - uniform, exponential and normal distributions
    - Central Limit Theorem
    - approximation of the binomial distribution using the normal distribution

- random vectors and joint distributions
    - covariance
    - correlation coefficient

- simulation in R

A4. Pareto distribution with $\alpha = 3$

PDF: $f(x) = \frac{3}{x^4}, x \geq 1$

CDF: $F(x) = \int_1^x f(t)dt = \int_1^x \frac{3}{t^4} dt = \frac{-1}{t^3}\Big|_1^x = -\frac{1}{x^3} + 1, \ x \geq 1$

$F^{-1}(x) = \frac{1}{\sqrt[3]{1-x}}$ $\longrightarrow$ inverse( ) from "GoFKernel" package

$y = 1 - \frac{1}{x^3} \iff \frac{1}{x^3} = 1 - y \iff x^3 = \frac{1}{1-y} \iff x = \frac{1}{\sqrt[3]{1-y}}$

A4. Pareto distribution with $a=3$

$$f(x)=\frac{3}{x^4}, \quad x\geq 1$$

$$F(x)=\int_{-\infty}^{x} f(t)\,dt=\int_{1}^{x} f(t)\,dt=\int_{1}^{x}\frac{3}{t^4}\,dt=-\frac{1}{t^3}\Big|_1^x=1-\frac{1}{x^3}$$

$$F^{-1}(x)=\frac{1}{\sqrt[3]{1-x}}, \quad x>1$$

$$y=1-\frac{1}{x^3} \Leftrightarrow \frac{1}{x^3}=1-y \Leftrightarrow x^3=\frac{1}{1-y} \Leftrightarrow x=\frac{1}{\sqrt[3]{1-y}}$$

There are three blocks,
floating in a sea of lava. Label the blocks 1; 2; 3, from left to right. Mark the Kangaroo, a video game character, is standing on block 1. To reach safety, he must get to block 3. He can't jump directly from block 1 to block 3; his only hope is to jump from block 1 to block 2, then jump from block 2 to block 3. Each time he jumps, he has probability 1/2 of success and probability 1/2 of "dying" by falling into the lava. If he "dies", he starts again at block 1.
Let J be the total number of jumps that Mark will make in order to get to block 3.

Write a function in R which simulates this game and returns the number of jumps Mark has to make each time to finish the game (in order to get to block 3).
Call your function 1000 times, store the results in a vector and compute the mean (average) of the values in that vector (average number of jumps Mark makes to get to block 3).

There are two urns with a total of 2N distinguishable balls. Initially, the fi rst urn has N white balls and the second urn has N black balls. At each stage, we pick a ball at random from each urn and interchange them. Let Xn be the number of black balls in the first urn at time n.

Write a function in R that simulates this game and returns the number Xn of black balls in the first urn.
The parameters of the function will be N, n.
Call the function for n=1000, 10000 and see what happens in the long run.