A_1 – Alice picks a class on Monday

A_2 – Tuesday

$\vdots$

A_5 – Friday

$$P(A_1 \cap A_2 \cap \ldots \cap A_5)$$

$$P(A_1 \cup A_2 \cup \ldots \cup A_5) = \sum_{i=1}^{\Omega} P(A_i) - \sum_{1 \le i < j \le 5} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \sum_{i<j<k<l} P(A_i \cap A_j \cap A_k \cap A_l) +$$

$$+ P(A_1 \cap A_2 \cap \ldots \cap A_5)$$

32)

R B B R

$\Omega = \{RRBB, RBRB, RBBR, BRBR, BRRB, BBRR\}$

$|\Omega| = 6$

$j=4$: RRBB $\qquad P(j=4) = \frac{1}{6}$

$j=0$: BBRR $\qquad P(j=0) = \frac{1}{6}$

$j=2$: RBRB, RBBR, BRBR, BRRB $\qquad P(j=1) = \frac{4}{6}$

$j=1,3 \rightarrow$ cannot happen

54) A_1 - Alice has a class on Monday
A_2 - Tuesday
$\vdots$
A_5 - Friday

$P(\bar{A}) = 1 - P(A)$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$P(A_1 \cap A_2 \cap \dots \cap A_5) = ?$

$$P(A_1 \cup A_2 \cup \dots \cup A_5) = \sum_{i=1}^{5} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \sum_{i<j<k<l} P(A_i \cap A_j \cap A_k \cap A_l)$$
$$+ P(A_1 \cap A_2 \cap \dots \cap A_5)$$

$$P(A_1 \cap A_2 \cap \dots \cap A_5) = 1 - P(\overline{A_1 \cap A_2 \cap \dots \cap A_5}) = \ ?$$

$$= 1 - P(\underset{B_1}{\underset{\parallel}{\bar{A}_1}} \cup \underset{B_2}{\underset{\parallel}{\bar{A}_2}} \cup \dots \cup \underset{B_5}{\underset{\parallel}{\bar{A}_5}})$$

$B_i = \overline{A_i}, \; i = \overline{1,5}$

$$P(B_1 \cup B_2 \cup \ldots \cup B_5) = \sum_{i=1}^{5} P(B_i) - \sum_{i<j} P(B_i \cap B_j) + \sum_{i<j<k} P(B_i \cap B_j \cap B_k)$$

B_i - Alice does not have a class on day i

$B_i \cap B_j \cap B_k \cap B_\ell = \phi$.

$$P(B_1 \cap B_2 \cap B_3) = \frac{2 \cdot \binom{6}{1} + 2 \cdot \binom{6}{2} \cdot \binom{6}{5} + 2 \cdot \binom{6}{3} \cdot \binom{6}{4}}{\binom{30}{7}} = \frac{12 + 2 \cdot 15 \cdot 6 + 2 \cdot 20 \cdot 15}{\binom{30}{7}} = \frac{12+180+600}{\binom{30}{7}} = \frac{792}{\binom{30}{7}}$$

Th	Fr
1	6
2	5
3	4
4	3
5	2
6	1

1, 6 w

2, 5 w

3, 4

$$P(B_i \cap B_j \cap B_k) = \frac{792}{\binom{30}{7}}$$

(31) N elks in the forest $\qquad N \longrightarrow m$

n - tagged elks $\qquad n \longrightarrow k$

m - sample size

$$P(k \text{ elks are tagged}) = \frac{\binom{n}{k} \cdot \binom{N-n}{m-k}}{\binom{N}{m}} \rightarrow \text{hypergeometric distribution}$$

$k = \overline{0, m}$

(28) $\underset{t_1}{\sqcup} \quad \underset{t_2}{\sqcup} \quad \dots \quad \underset{t_{10}}{\sqcup}$

$$P(\text{"at least 2 stats courses in the same time slot"}) = 1 - \frac{10 \cdot 9 \cdot 8}{10^3} = 0.28$$

2 cases:
- 2 stats courses in the same time slot: $10 \cdot 9 \cdot 3$
- all stats courses in the same time slot: 10

$$10 + 10 \cdot 9 \cdot 3 = 280$$

$$P = \frac{10 + 10 \cdot 9}{10^3} = \frac{280}{1000} = 0.28$$

34. (a) $P(\text{"flush"}) = \dfrac{4\cdot\left[\binom{13}{5}-1\right]}{\binom{52}{5}}$

54.

$\rightarrow$ case 1: $\dfrac{\text{1 day}}{\text{4 days}}$ with 3 lectures & with 1 lecture

$\rightarrow$ case 2: $\dfrac{\text{2 days}}{\text{3 days}}$ with 2 lectures & with 1 lecture

Case 1: $\binom{5}{1}\cdot\binom{6}{3}\cdot\binom{6}{1}^4 = 5\cdot\binom{6}{3}\cdot 6^4$

Case 2: $\binom{5}{2}\cdot\binom{6}{2}^2\cdot\binom{6}{1}^3 = \binom{5}{2}\binom{6}{2}^2\cdot 6^3$

Fav. outcomes $= 5\cdot\binom{6}{3}\cdot 6^4 + \binom{5}{2}\binom{6}{2}^2\cdot 6^3$

Total nr. outcomes $= \binom{30}{7}$

$P(\text{"Alice has classes every day"}) = \dfrac{5\cdot\binom{6}{3}\cdot 6^4 + \binom{5}{2}\binom{6}{2}^2\cdot 6^3}{\binom{30}{7}} = 0.3$

A_i - Alice has a class on day i, $i=\overline{1,5}$

$$P(\text{"Alice has a class everyday"}) = P(A_1 \cap A_2 \cap \ldots \cap A_5) = ?$$

$$P(\bar{A}) = 1 - P(A) \iff P(A) = 1 - P(\bar{A})$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_5) = 1 - P(\overline{A_1 \cap A_2 \cap \ldots \cap A_5}) = 1 - P(\bar{A}_1 \cup \bar{A}_2 \cup \ldots \cup \bar{A}_5) =$$
$$= 1 - P(B_1 \cup B_2 \cup \ldots \cup B_5)$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}, \quad \bar{A}_i = B_i, \ i=\overline{1,5}$$

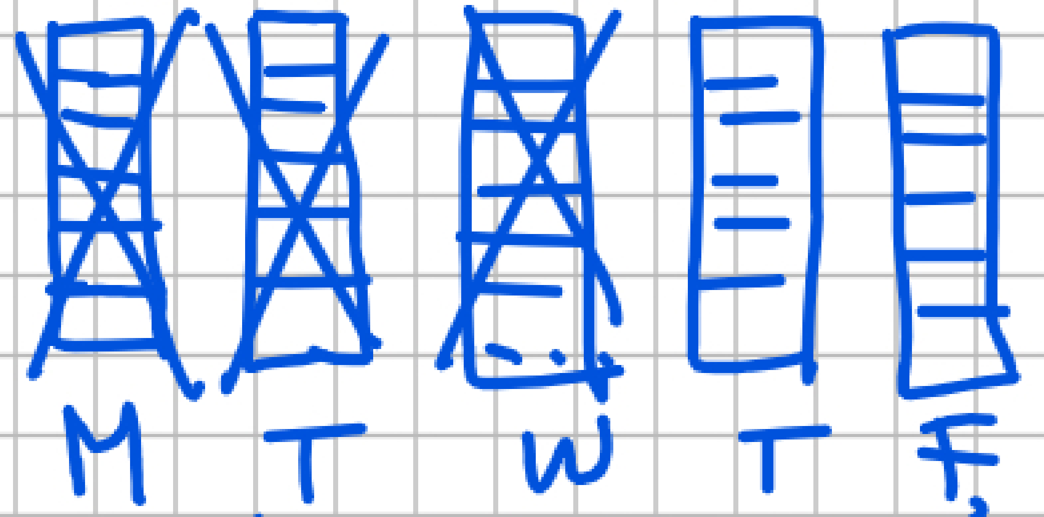
$$P(B_1 \cup B_2 \cup \ldots \cup B_5) = \sum_{i=1}^{5} P(B_i) - \sum_{i<j} P(B_i \cap B_j) + \sum_{i<j<k} P(B_i \cap B_j \cap B_k) -$$
$$- \sum_{i<j<k<l} \underbrace{P(B_i \cap B_j \cap B_k \cap B_l)}_{=0} + \underbrace{P(B_1 \cap B_2 \cap \ldots \cap B_5)}_{=0}$$

B_i: Alice does not have a class on day i (of the week), $i=\overline{1,5}$

$$P(B_1)=P(B_2)=\ldots=P(B_5)=\frac{\binom{24}{7}}{\binom{30}{7}}$$

$$P(B_i\cap B_j)=\frac{\binom{18}{7}}{\binom{30}{7}},\quad i\neq j,\quad i,j=\overline{1,5}$$

$$P(B_i\cap B_j\cap B_k)=\frac{\binom{12}{7}}{\binom{30}{7}},\quad i\neq j\neq k\neq i,\quad i,j,k=\overline{1,5}$$



$$\underbrace{\underline{30-6=24}}_{18=30-12}$$

$$P(B_1\cup B_2\cup\ldots\cup B_5)=5\cdot\frac{\binom{24}{7}}{\binom{30}{7}}-\underset{\binom{5}{2}}{\underset{\parallel}{10}}\cdot\frac{\binom{18}{7}}{\binom{30}{4}}+\binom{5}{3}\cdot\frac{\binom{12}{7}}{\binom{30}{7}}=\frac{\binom{5}{1}\cdot\binom{24}{7}-\binom{5}{2}\cdot\binom{18}{7}+\binom{5}{3}\cdot\binom{12}{7}}{\binom{30}{7}}=$$

$$=0.698$$

$$\left.\begin{matrix}1\\2\\3\\4\end{matrix}\right\}5\qquad \left.\begin{matrix}1\\2\\3\end{matrix}\right\}4\qquad \left.\begin{matrix}1\\2\end{matrix}\right\}3\qquad 1\}2$$

$$4+3+2+1=10$$

$$P(A_1\cap A_2\cap\ldots\cap A_5)=1-0.698=0.302$$

29 a) $\text{sum}=21$: $(6,5,5,5) \longrightarrow 4$

$(6,6,\underline{5},\underline{4}) \longrightarrow 4\cdot 3=12$

$(6,6,6,3) \longrightarrow 4$

$\underline{6}\ \underline{6}\ \underline{6}\ \underline{6}$

$5\cdot 4=\underline{20} \rightarrow$ at least one 6

$\text{sum}=22$: $(6,6,5,5) \longrightarrow 6 = \dfrac{4!}{2!\cdot 2!}$

$(6,6,6,4) \longrightarrow 4$

$$\frac{4!}{2!\,2!}=\frac{3\cdot 4}{2}=6$$

$P(\text{"sum}=21\text{"}) > P(\text{"sum}=22\text{"})$

$P(\text{"sum}=21\text{"}) = \dfrac{4+12+4}{6^4} = \dfrac{20}{6^4} = 0.015$

$P(\text{"sum}=22\text{"}) = \dfrac{10}{6^4}$

b) 2-letter palindrome : $\frac{26}{26^2} = \frac{1}{26}$

3-letter palindrome : $\frac{26^2}{26^3} = \frac{1}{26}$

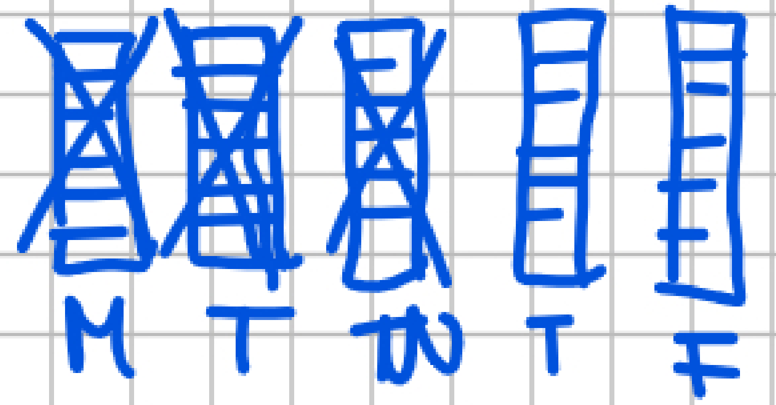
34) a) $P(\text{"flush"}) = \frac{4\cdot\left(\binom{13}{5}-1\right)}{\binom{52}{5}}$

$\frac{52!}{47!\cdot 5!} = \binom{52}{5}$

b) $P(\text{"two pair"}) = \frac{\binom{13}{2}\cdot\binom{4}{2}^2\cdot 44}{\binom{52}{5}}$

$52-8=44$

(54) $P(\text{"Alice has classes every day of the week"})$



$\rightarrow$ 7 classes

A_i - Alice does not have a class on day i (of the week), $i=\overline{1,5}$

$$P(\bar{A}_1 \cap \bar{A}_2 \cap \ldots \cap \bar{A}_5) = P(\overline{A_1 \cup A_2 \cup \ldots \cup A_5}) = 1 - P(A_1 \cup A_2 \cup \ldots \cup A_5) = 1 - 0.698 =$$

$$= \underline{\underline{0.302}}$$

$\downarrow$ "and"

$\bar{A} \cap \bar{B} = \overline{A \cup B}$, $P(\bar{A}) = 1 - P(A)$

Inclusion-exclusion formula:

$$P(A_1 \cup A_2 \cup \ldots \cup A_5) = \sum_{i=1}^{5} P(A_i) - \sum_{1 \le i < j \le 5} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \overset{0}{\overset{\|}{\sum_{i<j<k<l} P(A_i \cap A_j \cap A_k \cap A_l)}}$$

$$+ \underset{0}{\underset{\|}{P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5)}}$$

$$P(A_i) = \frac{\binom{24}{7}}{\binom{30}{7}}, \quad i = \overline{1,5}$$

$$P(A_i \cap A_j) = \frac{\binom{18}{7}}{\binom{30}{7}}, \quad i<j$$

$$P(A_i \cap A_j \cap A_k) = \frac{\binom{12}{7}}{\binom{30}{7}}$$

$$P(A_1 \cup A_2 \cup \dots \cup A_5) = 5 \cdot \frac{\binom{24}{7}}{\binom{30}{7}} - 10 \cdot \frac{\binom{18}{7}}{\binom{30}{7}} + 10 \cdot \frac{\binom{12}{7}}{\binom{30}{7}} = \frac{5 \cdot \binom{24}{7} - 10 \cdot \binom{18}{7} + 10 \cdot \binom{12}{7}}{\binom{30}{7}} =$$

$$= 0.698$$

1,2	2,3	3,4	4,5
1,3	2,4	3,5	
1,4	2,5		
1,5			

$$10 = \binom{5}{2} = \binom{5}{3}$$

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| + \dots + (-1)^{n+1} |A_1 \cap A_2 \cap \dots \cap A_n|$$

→ 2 remaining balls

7 balls (classes)

2 cases : 1) both balls in one box (one day)

2) put the balls in 2 different boxes (2 days)

$\boxed{C1}$ $P_1 = \dfrac{\binom{5}{1}\cdot\binom{6}{3}\cdot\binom{6}{1}^4}{\binom{30}{7}}$

$\boxed{C2}$ $P_2 = \dfrac{\binom{5}{2}\cdot\binom{6}{2}^2\cdot\binom{6}{1}^3}{\binom{30}{7}}$

day with 3 lectures

$\binom{6}{3}$ - choose the lectures out of 6 available lectures

$P = P_1 + P_2$

37) a) C1: $p_1 = \frac{4}{52}$

$P = p_1 + p_2 + \ldots + p_{37}$?

C2: $\square$ A $\quad p_2 = \frac{36\cdot 4}{52\cdot 51}$

C3: $\square\square$ A $\quad p_3 = \frac{36\cdot 35\cdot 4}{52\cdot 51\cdot 50}$

C4: $\square\square\square$ A $\quad p_4 = \frac{36\cdot 35\cdot 34\cdot 4}{52\cdot 51\cdot 50\cdot 49}$

$\vdots$

C_{37} $\quad p_{37} = \frac{36\cdot 35\cdot\ldots\cdot 2\cdot 1\cdot 4}{52\cdot 51\cdot\ldots\cdot 16}$.

$$p_k = \frac{36\cdot 35\cdot\ldots\cdot(36-k+2)\cdot 4}{52\cdot 51\cdot\ldots\cdot(52-k+1)} \rightarrow \underline{k\geq 2}$$

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = x_1 \cdot \begin{vmatrix} y_2 & 1 \\ y_3 & 1 \end{vmatrix} - x_2 \cdot \begin{vmatrix} y_1 & 1 \\ y_3 & 1 \end{vmatrix} + x_3 \cdot \begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} =$$

$$= x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)$$

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| - \ldots + (-1)^{n+1} \cdot |A_1 \cap A_2 \cap \ldots \cap A_n|$$

$n=2$, $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \overset{\text{and}}{\cap} A_2)$

$n=3$, $P(A_1 \underset{\text{or}}{\cup} A_2 \cup A_3) = \sum_{i=1}^{3} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + P(A_1 \cap A_2 \cap A_3)$

$$= P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

A_i - Alice has a class on day i (of the week), $i=\overline{1,5}$

$P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = ?$

$B_i = \overline{A_i}$: Alice does not have a class on day i, $i=\overline{1,5}$

$P(\overline{A}) = 1 - P(A) \iff P(A) = 1 - P(\overline{A})$

$P(A_1 \cap A_2 \cap \dots \cap A_5) = 1 - P(\overline{A_1 \cap A_2 \cap \dots \cap A_5}) = 1 - P(\overline{A}_1 \cup \overline{A}_2 \cup \dots \cup \overline{A}_5) =$

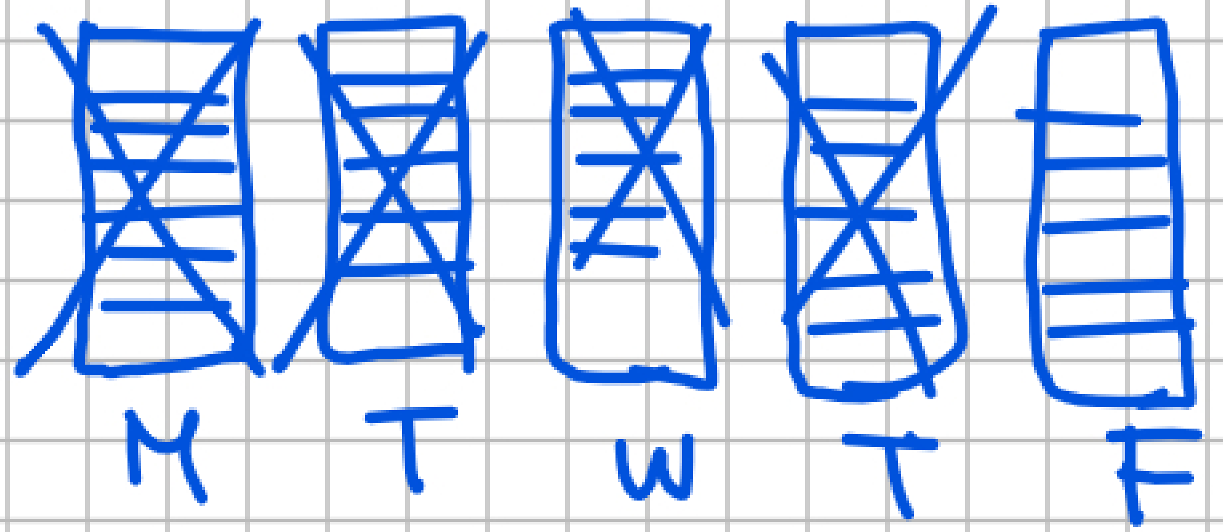
$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$= 1 - P(B_1 \cup B_2 \cup \dots \cup B_5) = 1 - 0.698 = \underline{0.302}$

$P(B_1 \cup B_2 \cup \dots \cup B_5) = \sum_{i=1}^{5} P(B_i) - \sum_{i<j} P(B_i \cap B_j) + \sum_{i<j<k} P(B_i \cap B_j \cap B_k) - \sum_{i<j<k<l} \underbrace{P(B_i \cap B_j \cap B_k \cap B_l)}_{= 0}$

$+ \underbrace{P(B_1 \cap B_2 \cap \dots \cap B_5)}_{= 0}$

$$P(B_i)=\frac{\binom{24}{7}}{\binom{30}{7}},\quad i=\overline{1,5}$$



7

$$P(B_i\cap B_j)=\frac{\binom{18}{7}}{\binom{30}{7}},\quad i,j=\overline{1,5},\ i<j$$

$$P(B_i\cap B_j\cap B_k)=\frac{\binom{12}{7}}{\binom{30}{7}},\quad i<j<k,\ i,j,k=\overline{1,5}$$

$$P(B_1\cup B_2\cup\dots\cup B_5)=5\cdot\frac{\binom{24}{7}}{\binom{30}{7}}-\overset{\binom{5}{2}}{\overset{\shortparallel}{10}}\cdot\frac{\binom{18}{7}}{\binom{30}{7}}+\overset{\binom{5}{3}}{\overset{\shortparallel}{10}}\cdot\frac{\binom{12}{7}}{\binom{30}{7}}=0.698$$

1,2   2,3   3,4   4,5
1,3   2,4   3,5
1,4   2,5
1,5

$i<j$

1,2,3   1,4,5   3,4,5
1,2,4   2,3,4
1,2,5   2,3,5
1,3,4   2,4,5
1,3,5

(23) $F_1 \quad F_2 \quad F_3 \quad F_4 \quad \dots \quad F_8 \quad F_9 \quad F_{10}$

$$P(\text{"3 consecutive floors"}) = \frac{7}{\binom{9}{3}} \approx 0.083$$

$2-8: \ 8-2+1=7$

(24) $B_1, B_2, B_3, G_1, G_2, G_3$

$$P(\text{"the 3 eldest children are the 3 girls"}) = \frac{3! \cdot 3!}{6!} = \frac{6}{4 \cdot 5 \cdot 6} = \frac{1}{20} \approx 0.05$$

$\underbrace{G_1, \ G_2, \ G_3}\ \underbrace{B_2, \ B_1, \ B_3}$

$G_1 \quad G_3 \quad G_2$

$G_2 \quad G_1 \quad G_3$

$\vdots$