Lab 14. sgr. 6

Exercise 101. Air pollution is determined by measuring several different elements that can be detected in the air. One of them is carbon monoxide. The following sample of daily readings was obtained from a local newspaper:

3.5 3.9 2.8 3.1 3.1 3.4 4.8 3.2 2.5 3.5 4.4 3.1

a. Compute the mean and the standard deviation of the sample.

b. Carbon monoxide is measured and interpreted according to the accompanying scale. Does the sample show sufficient evidence to allow us to conclude that the carbon level monoxide is low, that is, $\mu < 4.9$ at $\alpha = 0.05$?

c. Does the sample show sufficient evidence to allow us to reject the claim that the variance in the carbon monoxide readings is no more than 0.25 at $\alpha = 0.05$? H_0:

d. Construct the 90%, 95% and 99% confidence intervals for estimating the mean daily level of carbon monoxide pollution. μ

e. Construct the 90%, 95% and 99% confidence intervals for estimating the standard deviation of carbon monoxide pollution.

one-sample t-test (σ is unknown)

$t_\alpha = qt(\alpha, n-1)$

$t_\alpha = -1.79$

$R = (-\infty, -1.79)$

$T = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = -7.73$

$T \in R \Rightarrow H_0$ is rejected

At the 5% level of signif. the sample shows sufficient evidence to say the level of carbon monoxide is low.

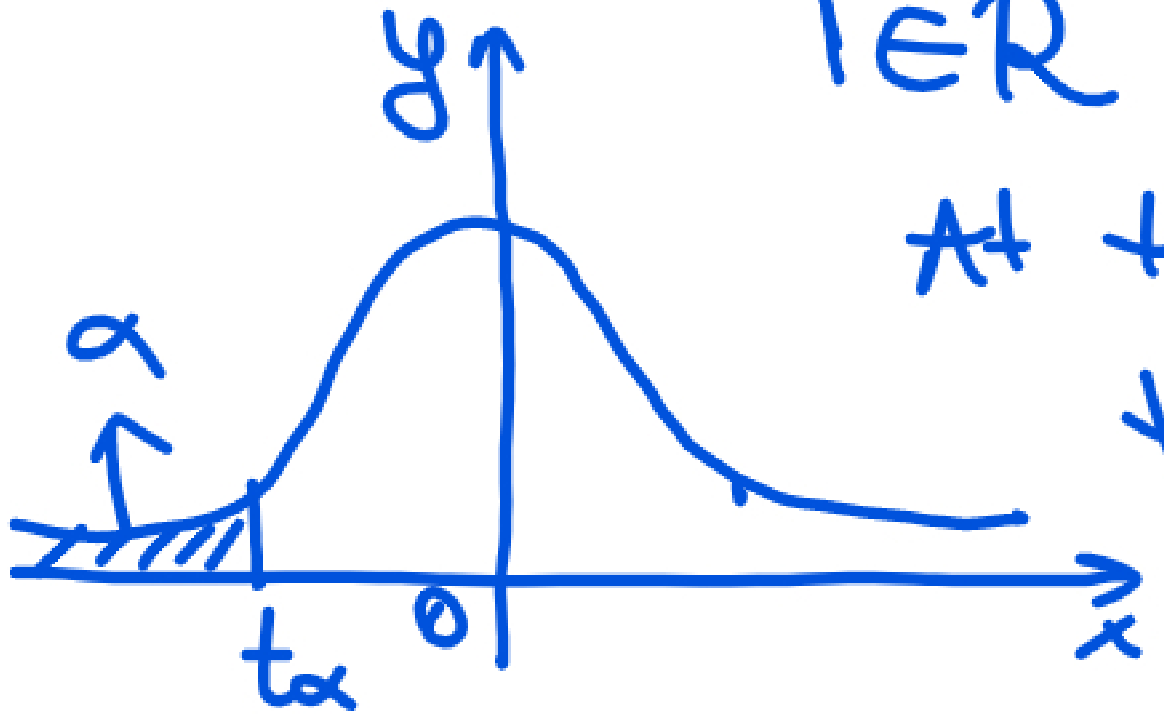
a) $\bar{x} = 3.44$

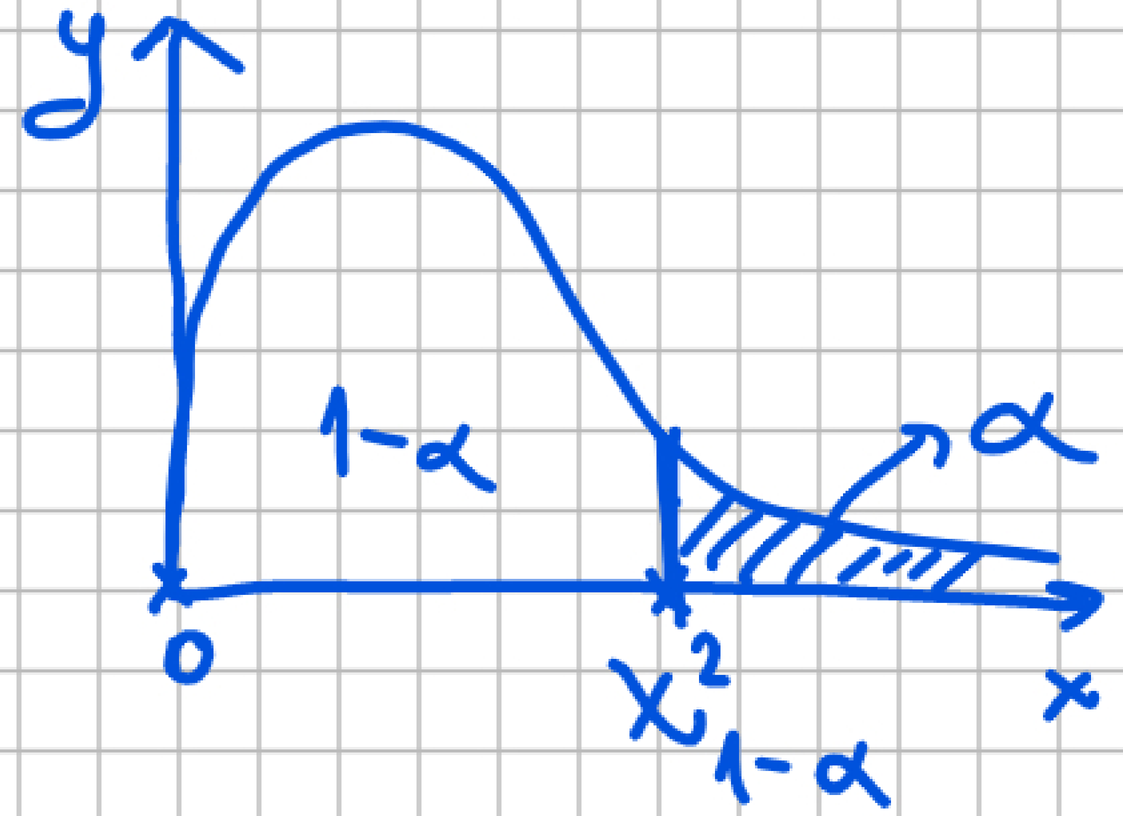
$s = 0.65$

b) $H_0: \mu = 4.9$ $(\mu_0 = 4.9)$

$H_a: \mu < 4.9 \Rightarrow R = (-\infty, t_\alpha)$

$\alpha = 0.05$





c) $H_0: \sigma^2 = 0.25$ $\quad (\sigma_0^2 = 0.25)$

$H_a: \sigma^2 > 0.25 \Rightarrow R = (\chi^2_{1-\alpha}, \infty)$

χ^2 test for variance

$\chi^2_{1-\alpha}$ = qchisq(1-α, n-1) = qchisq(α, n-1, lower.tail=F) = 4.57 $\Rightarrow R = (4.57, \infty)$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = 18.75 \Rightarrow \chi^2 \in R \Rightarrow H_0 \text{ is rejected}$$

$n = 12$

s^2 = sample variance

$s^2 = 0.65^2$

At the 5% level of significance, the sample shows sufficient evidence to reject the claim that the variance in the carbon monoxide readings is no more than 0.25.

d) $(1-\alpha)100\%$ ci for μ : $\left[\bar{x} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{m}}, \bar{x} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{m}}\right]$

$\alpha = 0.1 \Rightarrow -t_{\frac{\alpha}{2}} = qt(\alpha/2, m-1) = -1.79 \Rightarrow t_{\alpha/2} = 1.79$

$\alpha = 0.05 \Rightarrow -t_{\alpha/2} = qt(\alpha/2, m-1) = -2.2 \Rightarrow t_{\alpha/2} = 2.2$

$\alpha = 0.01 \Rightarrow -t_{\alpha/2} = qt(\alpha/2, m-1) = -3.1 \Rightarrow t_{\alpha/2} = 3.1$

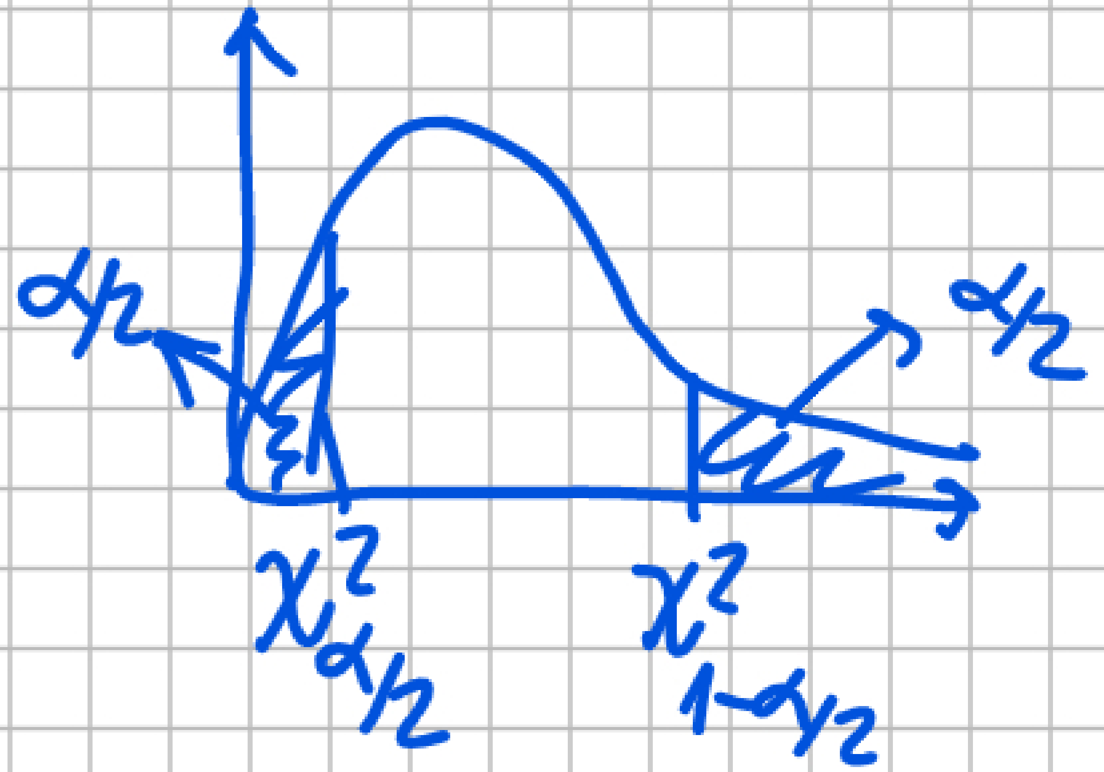
$\bar{x} = 3.44, s = 0.65, m = 12$

90% ci for μ: $[3.10, 3.78]$

95% ci for μ: $[3.02, 3.85]$

99% ci for μ: $[2.85, 4.02]$

e) $(1-\alpha)100\%$ ci for σ: $\left[\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}}\right]$



$\alpha=0.1 \Rightarrow \chi^2_{\alpha/2}=4.57 \quad \chi^2_{1-\alpha/2}=19.67$

$\alpha=0.05 \Rightarrow \chi^2_{\alpha/2}=3.81 \quad \chi^2_{1-\alpha/2}=21.92$

$\alpha=0.01 \Rightarrow \chi^2_{\alpha/2}=2.60 \quad \chi^2_{1-\alpha/2}=26.75$

$\chi^2_{\frac{\alpha}{2}}=$ qchisq$(\alpha/2, n-1)$

$\chi^2_{1-\alpha/2}=$ qchisq$(1-\alpha/2, n-1)$

90% ci for σ: $[0.48, 1.01]$

95% ci for σ: $[0.46, 1.1]$

99% ci for σ: $[0.41, 1.34]$

Exercise 107. For a sample of 10 students, the following bivariate data represents the distance and the duration of their travel to school.

x	1	3	5	5	7	7	8	10	10	12
y	5	10	15	20	15	25	20	25	35	35

1. Determine the scatter diagram and the correlation coefficient of the sample.
2. Does this sample show sufficient evidence for the positive linear correlation of the distance and the duration of travel in the case of all students ?
3. Find the equation of the regression line.
4. Does the slope b_1 of the regression line show sufficient evidence to claim that $\beta_1 > 0$ at a significance level $\alpha = 0.05$?

sgr. 1

Exercise 102. It has been suggested that abnormal human males tend to occur more in children born to older-than-average parents. Case histories of 20 abnormal males were obtained and the ages of 20 mothers were:

31 21 29 28 34 45 21 41 27 31 43 21 39 38 32 28 37 28 16 39

The mean age at which mothers in the general population give birth is 28 years.

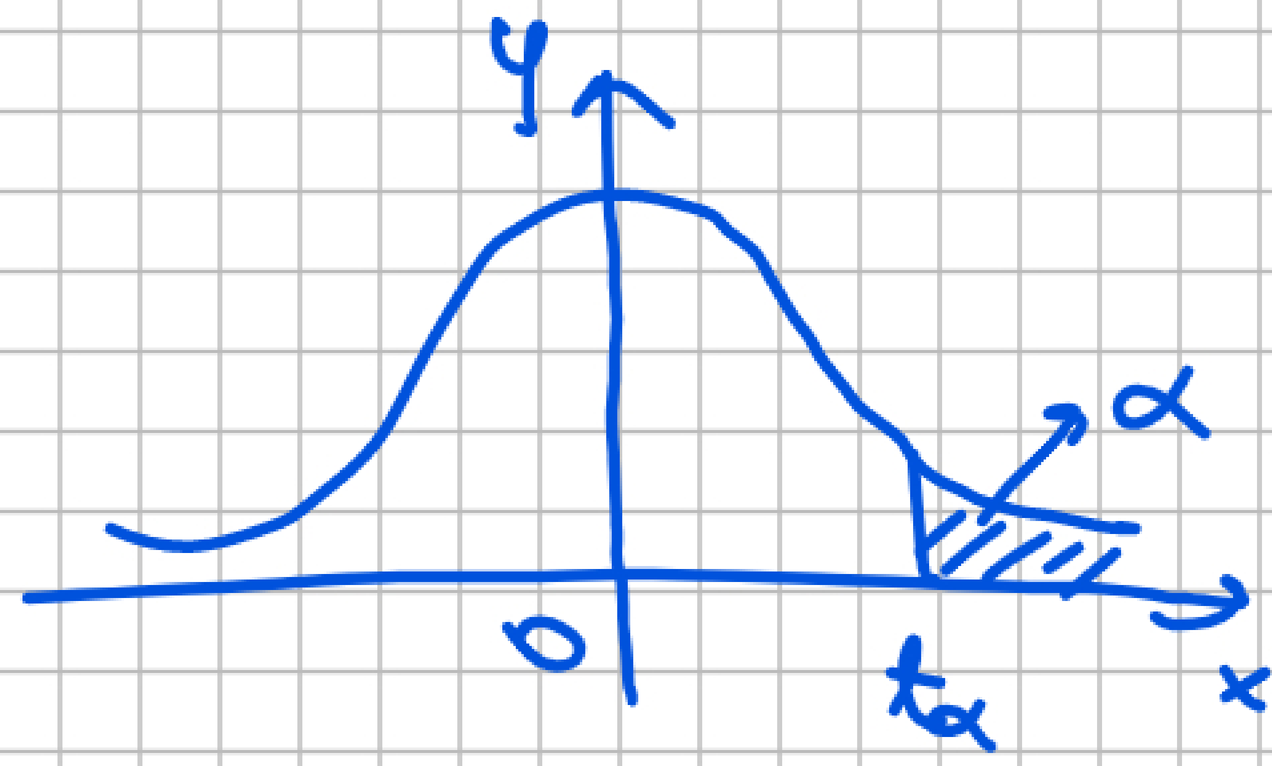
a. Compute the mean and the standard deviation of the sample.

b. Does the sample show sufficient evidence to support the claim that abnormal male children have older-than-average mothers? Use $\alpha = 0.05$.

c. Compute the 95% confidence interval for estimating the standard deviation of the ages of all mothers for these children.

d. Does the sample show sufficeint evidence to support the claim that the standard deviation of mothers' ages of abnormal males is less than 8.5?

e. Compute the 90% confidence interval for the standard deviation of mothers' ages.



$t_\alpha = qt(1-\alpha, n-1) = 1.73$

a) $\bar{X} = 31.45$

$s = 8.05$

b) $H_0: \mu = 28$ $(\mu_0 = 28)$ $n = 20$

$H_a: \mu > 28 \Rightarrow R = (t_\alpha, \infty)$

one-sample t-test (σ is unknown)

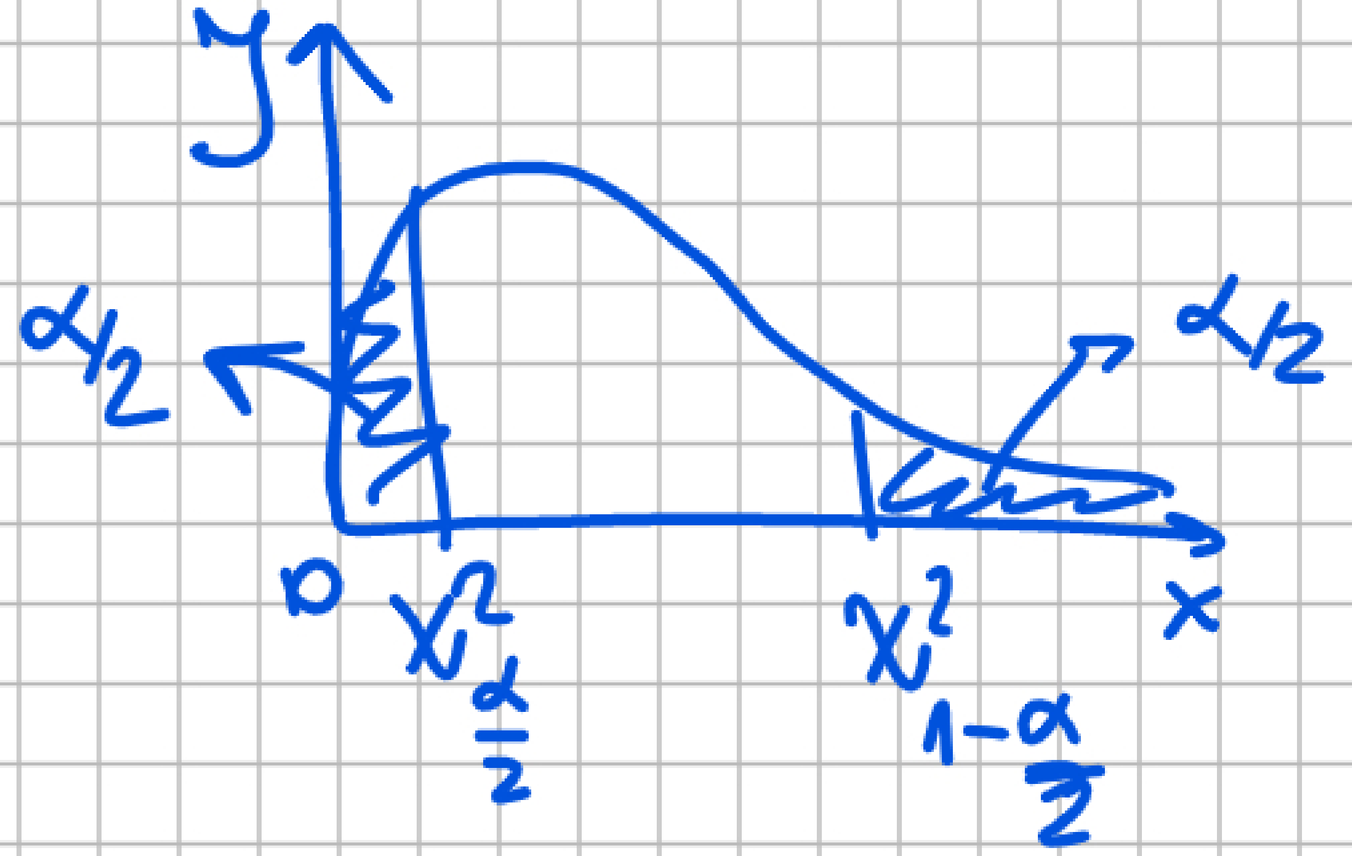
$T = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$

$T = 1.92$

$R = (1.73, \infty)$

$T \in R \Rightarrow H_0$ is rejected

At the 5% level of significance, the sample shows sufficient evidence to support the claim that abnormal moles have older-than-average mothers.



t.test(age, mu=28, alternative="greater")

p-value $= 0.035 < \alpha = 0.05 \Rightarrow H_0$ is rejected

95%

e) $(1-\alpha)100\%$ CI for σ: $\left[\sqrt{\dfrac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}}, \sqrt{\dfrac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}}}\right] = [6.12, 11.76]$

$s^2 = 64.8$, $n = 20$, $\chi^2_{\frac{\alpha}{2}} =$ qchisq$(\alpha/2, n-1) = 8.91$, $\chi^2_{1-\frac{\alpha}{2}} =$ qchisq$(1-\alpha/2, n-1) = 32.85$

d) $H_0: \sigma^2 = 8.5^2 \quad (\sigma_0 = 8.5)$

$H_a: \sigma^2 < 8.5^2 \Rightarrow R = (0, \chi^2_\alpha)$

χ^2 test for variance / std. deviation

$\chi^2_\alpha = \text{qchisq}(\alpha, n-1) = 10.12$

$R = (0, 10.12)$

$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = 17.04$

$n = 20, \ s^2 = 64.79$

$\chi^2 \notin R \Rightarrow H_0$ is not rejected

At the 5% level of significance the sample doesn't show sufficient evidence to support the claim that the std. deviation of mothers' ages is less than 8.5.

EnvStats package → install it

library(EnvStats)

varTest(age, sigma.squared = 8.5^2, alternative = "less")

p-value $= 0.41 > \alpha = 0.05 \Rightarrow H_0$ is not reject