Example 2. We have to accept or reject a large shipment of items. For quality control purposes, we collect a sample of 200 items and find 24 defective items in it. The manufacturer claims that at most one in 10 items in the shipment is defective. At the 1% level of significance, do we have sufficient evidence to disprove this claim?

$n = 200$ (sample size)

24 items are defective

$\hat{p} = \frac{24}{200} = \frac{3}{25}$ – proportion of defective items (sample proportion)

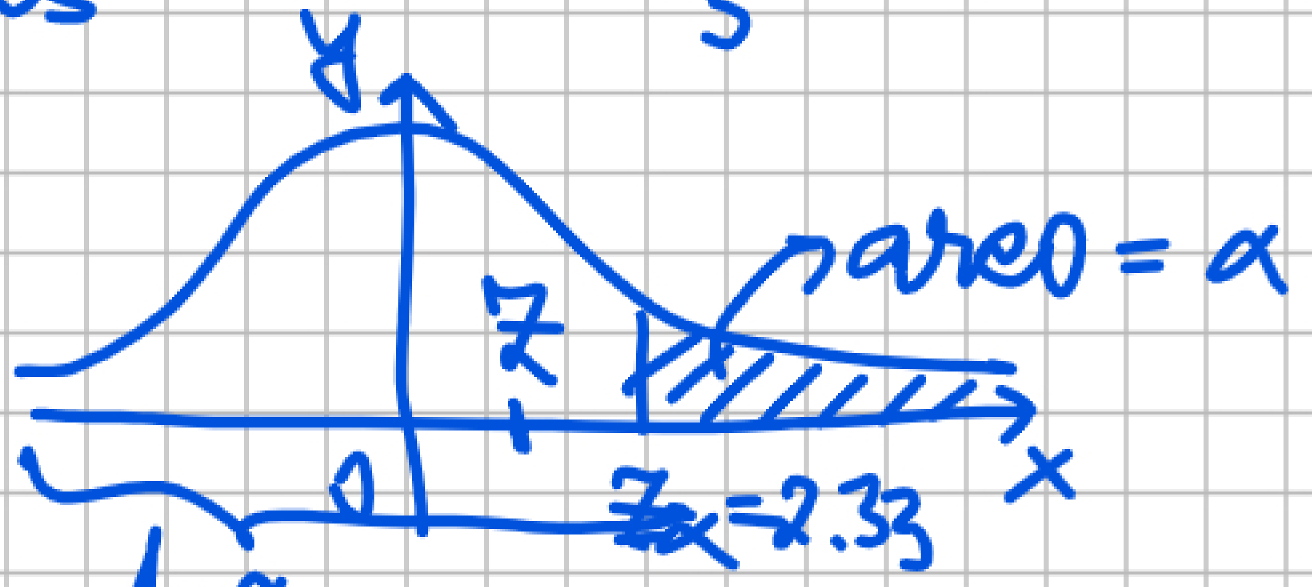
$H_0: p = \frac{1}{10}$ $(= 0.1)$ $\rightarrow p_0 = \frac{1}{10}$

$H_a: p \boxed{>} \frac{1}{10}$ $\Rightarrow R = (z_\alpha, \infty)$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sim N(0,1)$$

$$z = \frac{\frac{3}{25} - \frac{1}{10}}{\sqrt{\frac{1}{10} \cdot \frac{9}{10} \cdot \frac{1}{200}}} = \frac{\frac{1}{50}}{\frac{3}{100\sqrt{2}}} = \frac{100\sqrt{2}}{150} = \frac{2\sqrt{2}}{3}$$

$$z = \frac{2\sqrt{2}}{3} = 0.94$$



z_α = qnorm$(1-\alpha)$ = qnorm$(0.99) = 2.33$

$\alpha = 1\% = 0.01$

$Z = 0.94 \notin R = (2.33, \infty) \Rightarrow H_0$ is not rejected

→ the manufacturer's claim is true (we don't have sufficient evidence to disprove the claim) → z.test() in the BSDA package

Example 3. A quality inspector finds 10 defective parts in a sample of 500 parts received from manufacturer A. Out of 400 parts from manufacturer B, she finds 12 defective ones. A computer-making company uses these parts in their computers and claims that the quality of parts produced by A and B is the same. At the 5% level of significance, do we have enough evidence to disprove this claim?

- 2 populations: parts from A and parts from B
- 2 samples: 500 parts from A ($n = 500$)
  400 parts from B ($m = 400$)
- sample proportions of defective items: $\hat{p}_1 = \frac{1\cancel{0}}{50\cancel{0}}$, $\hat{p}_2 = \frac{12}{400} = \frac{3}{100}$
- $\alpha = 0.05$

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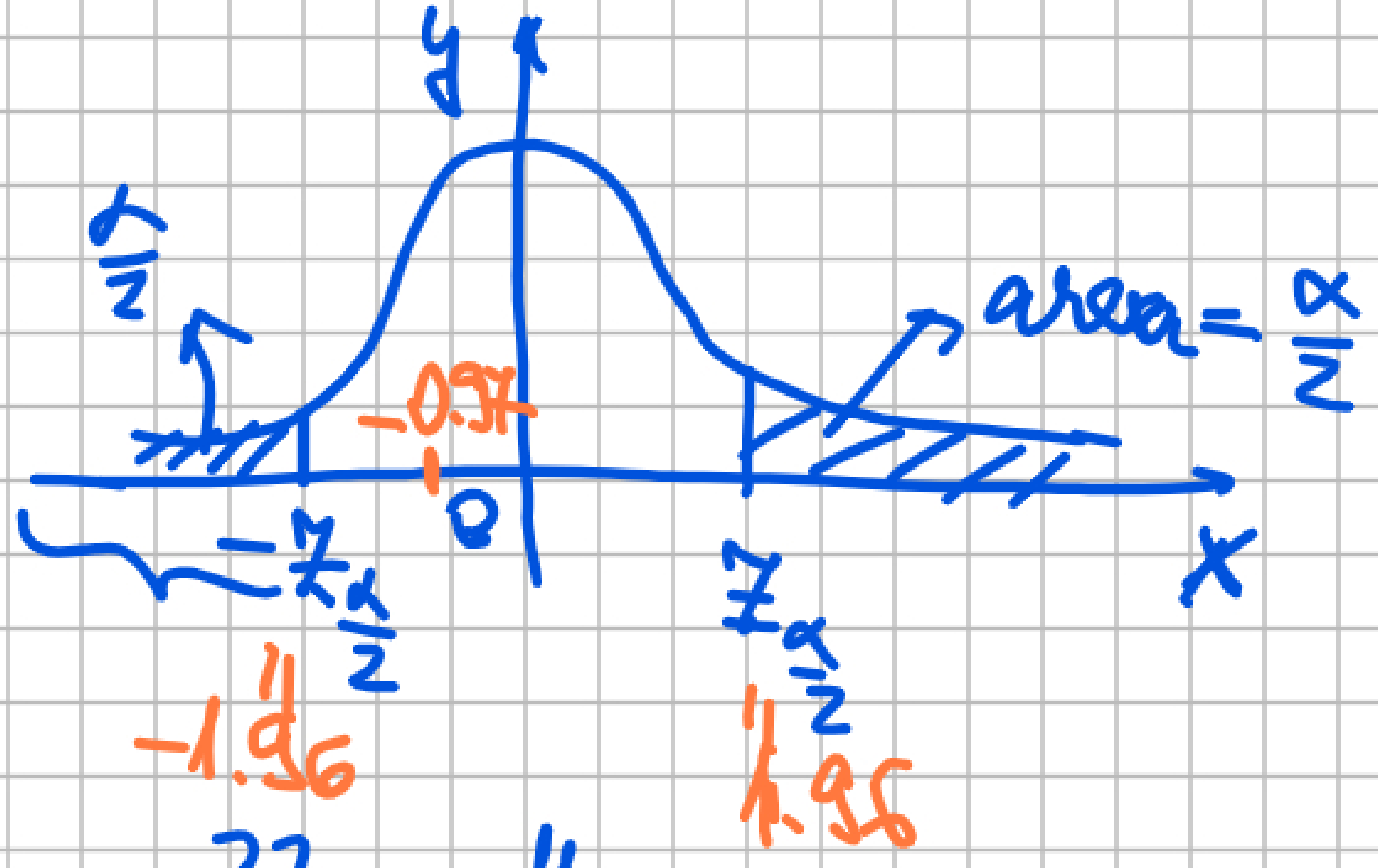
$H_0: p_1 = p_2$

$H_a: p_1 \neq p_2 \Rightarrow R = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, \infty)$



• p_1, p_2 – population proportions

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n} + \frac{1}{m})}}$$

$$\hat{p} = \frac{m\hat{p}_1 + m\hat{p}_2}{m+m} = \frac{10+12}{500+400} = \frac{22}{900} = \frac{11}{450}$$

$$Z = \frac{0.02 - 0.03}{\sqrt{\frac{11}{450} \cdot \frac{439}{450} \cdot \left(\frac{1}{500} + \frac{1}{400}\right)}} = \frac{-0.01}{\frac{1}{450} \cdot \sqrt{439 \cdot 11 \cdot \frac{900}{400 \cdot 500}}} = \frac{-0.01 \cdot 450}{\frac{30}{200} \cdot \sqrt{\frac{439 \cdot 11}{5}}} = \frac{-0.01 \cdot \overset{15}{\cancel{450}} \cdot 200\sqrt{5}}{\cancel{30} \cdot \sqrt{11 \cdot 439}} \Leftrightarrow$$

$$Z = -\frac{30\sqrt{5}}{\sqrt{11 \cdot 439}} = -0.97$$

$$-z_{\alpha/2} = \text{qnorm}(\alpha/2) = -1.96 \Rightarrow z_{\alpha/2} = 1.96 \Rightarrow R = (-\infty, -1.96) \cup (1.96, \infty)$$

$\Rightarrow Z \notin R \Rightarrow$ we cannot reject H_0

(we don't have enough evidence to disprove the claim)

(the quality is the same)

Example 4. (Unauthorized use of a computer account, continued). A long-time authorized user of the account makes 0.2 seconds between keystrokes. One day, the data in Example 9.19 on p. 260 are recorded as someone typed the correct username and password. At a 1% level of significance, is this an evidence of an unauthorized attempt?

.24, .22, .26, .34, .35, .32, .33, .29, .19, .36, .30, .15, .17, .28, .38, .40, .37, .27

$\alpha = 0.01$

$H_0: \mu = 0.2$

$H_a: \mu \neq 0.2 \Rightarrow R(-\infty, -t_{\alpha/2}) \cup (t_{\frac{\alpha}{2}}, \infty)$

$(\mu > 0.2) \Rightarrow R = (t_\alpha, \infty)$

- population: time between keystrokes for a user
- sample size: $n = 18$
- μ - population mean (average time between keystrokes)

$T = \dfrac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}}$ - has a Student t distribution with 17 degrees of freedom

$\mu_0 = 0.2$

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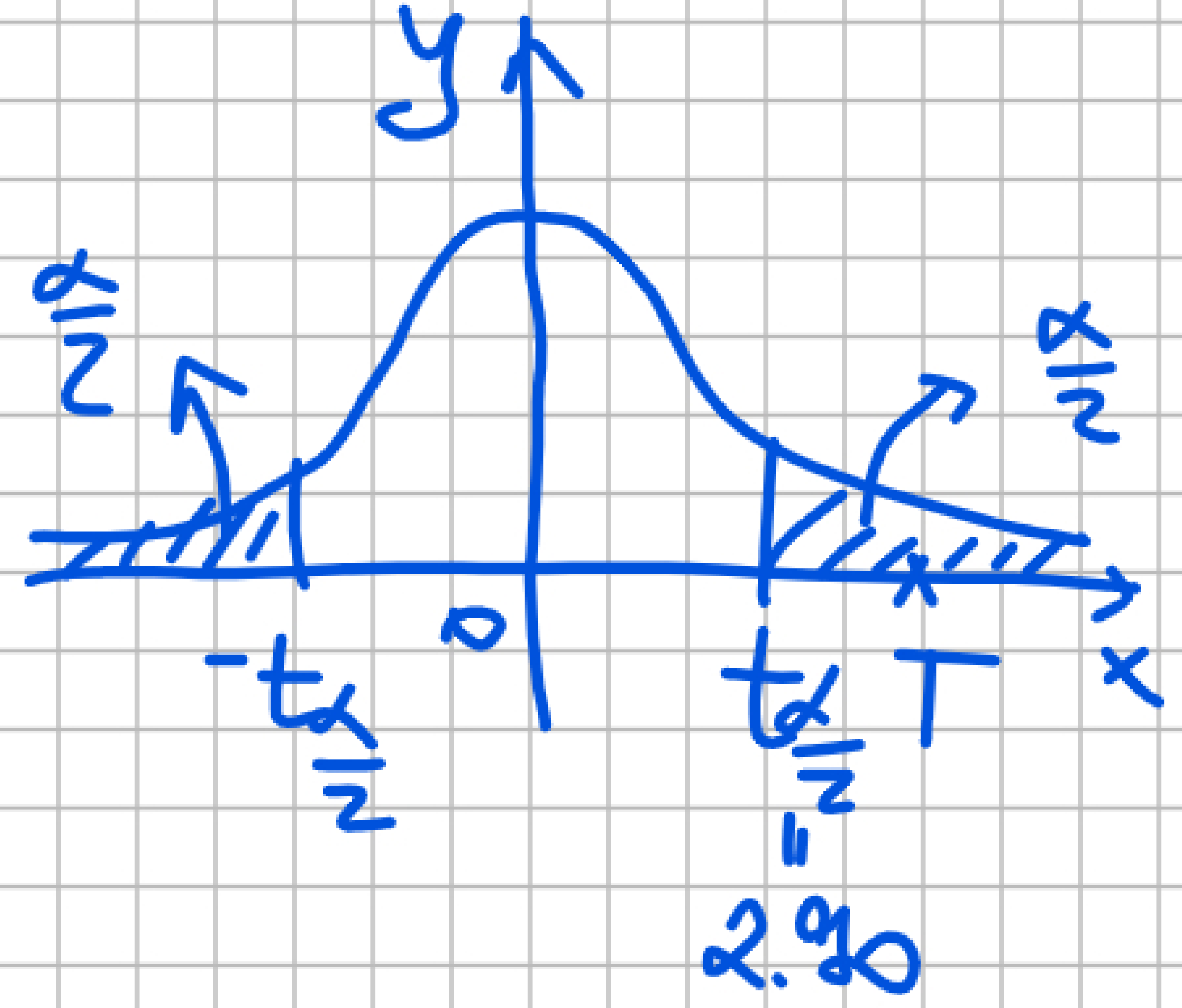
$$\bar{X} = \frac{0.24+0.22+\dots+0.27}{18} = 0.29$$

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \sqrt{\frac{1}{17}\sum_{i=1}^{18}(x_i-\bar{x})^2} = 0.07$$



$$T = \frac{0.29-0.2}{\frac{0.07}{\sqrt{18}}} = \frac{0.09\cdot 3\sqrt{2}}{0.07396} = \frac{27\sqrt{2}}{7.396} = 5.45, \quad (\underline{\underline{5.16}})$$

$$-t_{\frac{\alpha}{2}} = qt(\alpha/2, 17) = -2.90 \Rightarrow t_{\alpha/2} = 2.90$$

$$R = (-\infty, -2.90) \cup (2.90, \infty)$$

$\Rightarrow T \in R \Rightarrow H_0$ is rejected (the user is an unauthorized one)

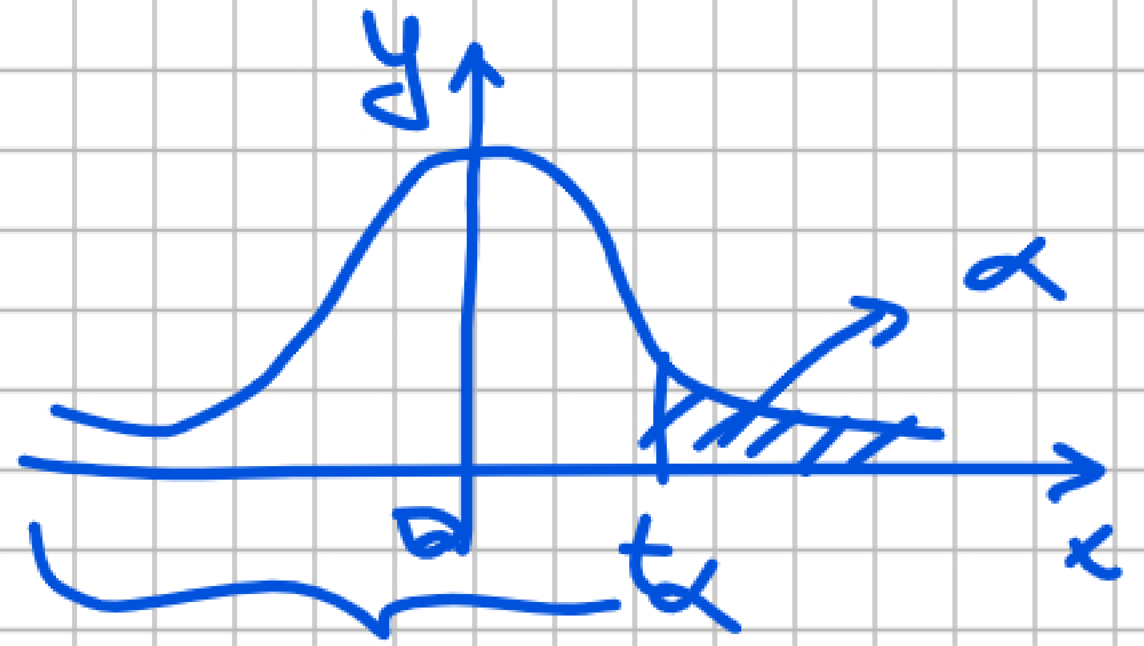
- $H_0: \mu = 0.2$

  $H_a: \mu > 0.2 \Rightarrow R = (t_\alpha, \infty)$

  $t_\alpha = qt(1-\alpha, 17) = 2.57$

  $\Rightarrow R = (2.57, \infty)$

  $T = 5.45 \in R \Rightarrow H_0$ is reject



In R: t.test()

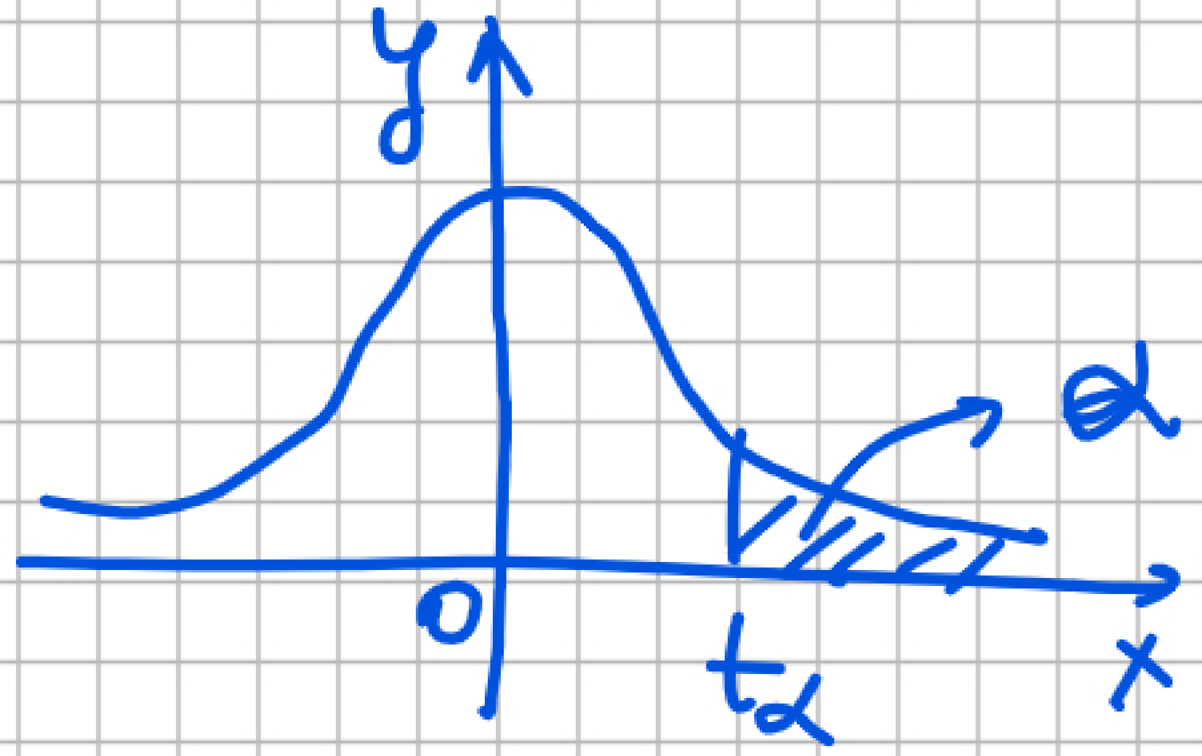
t.test(time, mu= 0.2, conf.level= 0.99)

p-value $= 7.81 \cdot 10^{-5} \ll \alpha = 0.01 \Rightarrow H_0$ is rejected

p-value $< \alpha \Rightarrow H_0$ is rejected

$> \alpha \Rightarrow H_0$ is not rejected

Example 5. (CD writer and battery life). Does a CD writer consume extra energy, and therefore, does it reduce the battery life on a laptop? The data collected is the following: eighteen users without a CD writer worked an average of 5.3 hours with a standard deviation of 1.4 hours; other twelve, who used their CD writer, worked an average of 4.8 hours with a standard deviation of 1.6 hours. Consider a level of significance $\alpha = 0.1$



- $n = 18$, $m = 12$ (the 2 sample sizes)

$\bar{X} = 5.3$

$\bar{Y} = 4.8$

$s_x = 1.4$, $s_y = 1.6$

$\alpha = 0.1$

$\sigma_1 = \sigma_2$

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2 \Rightarrow R = (t_\alpha, \infty)$

$$T = \frac{\bar{X} - \bar{Y}}{s_p \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}}$$ has a Student t distribution with $n+m-2$ df

$$s_p = \sqrt{\frac{1}{n+m-2}\left((n-1)s_x^2 + (m-1)s_y^2\right)} = 1.48$$

$$T = \frac{5.3 - 4.8}{s_p \cdot \sqrt{\frac{1}{18} + \frac{1}{12}}} = 0.91$$

$t_\alpha = qt(1-\alpha, \underbrace{18+12-2}_{28}) = 1.31$

$T = 0.91 \notin R = (1.31, \infty) \Rightarrow$ we cannot reject H_0

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9.10. We have to accept or reject a large shipment of items. For quality control purposes, we collect a sample of 200 items and find 24 defective items in it.

(a) Construct a ~~96%~~ 95% confidence interval for the proportion of defective items in the whole shipment.

(b) The manufacturer claims that at most one in 10 items in the shipment is defective. At the ~~4%~~ 5% level of significance, do we have sufficient evidence to disprove this claim? Do we have it at the ~~15%~~ 1% level?

$\alpha = 5\% = 0.05$

$\alpha = P(\text{type I error})$

$= P(H_0 \text{ is rejected} / H_0 \text{ is true})$

$n = 200$ (sample size)

24 defective items

$\alpha = 0.05$

$H_0: p = \frac{1}{10}$ $\left(p_0 = \frac{1}{10}\right)$

$H_a: p \neq \frac{1}{10} \Rightarrow R = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, \infty)$

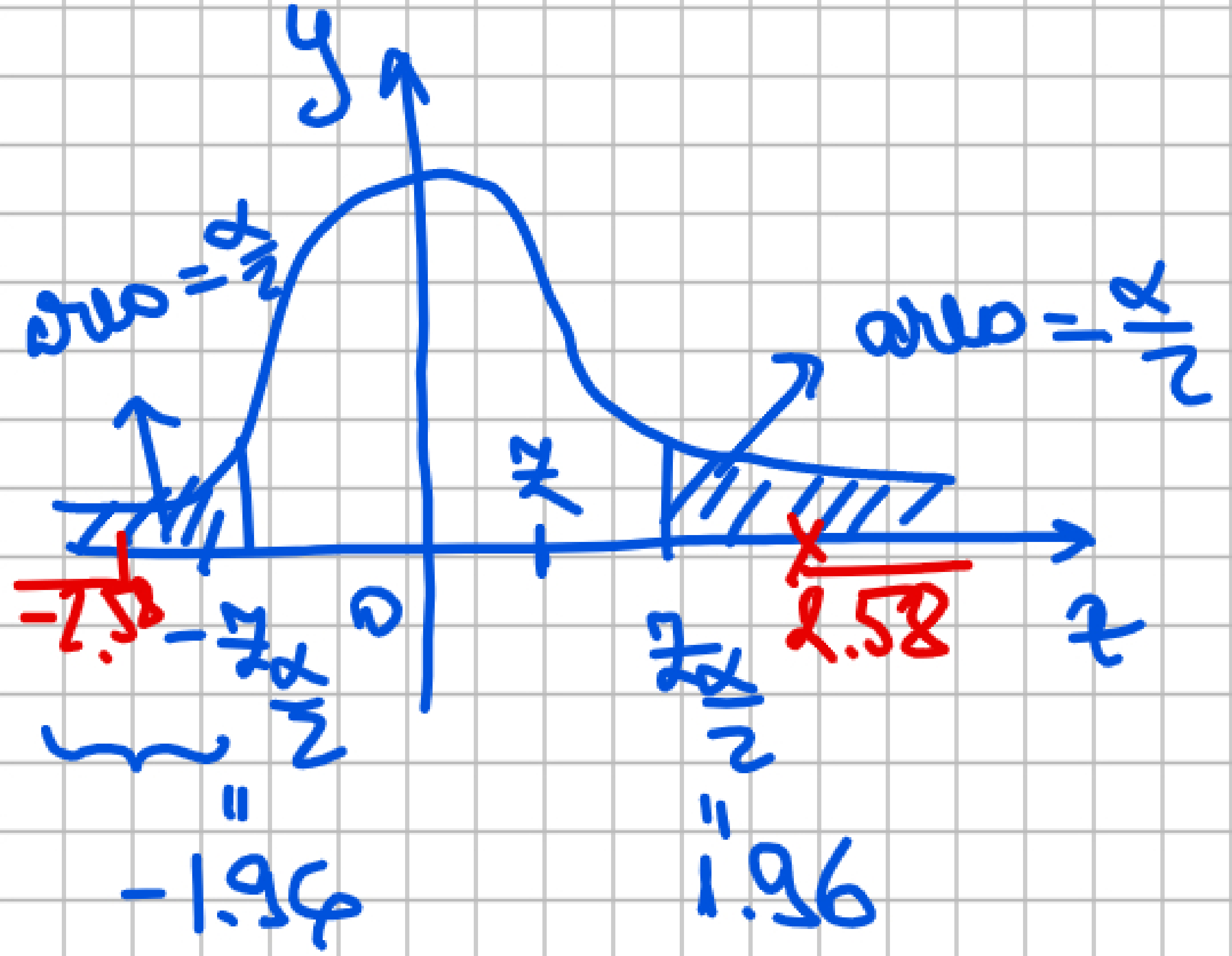
$p > \frac{1}{10} \Rightarrow R = (z_\alpha, \infty)$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sim N(0,1)$$

$\hat{p}$ – sample proportion (of defective items)

$\hat{p} = \frac{24}{200} = \frac{3}{25}$, $Z = \frac{0.12 - 0.10}{\sqrt{\frac{0.10 \cdot 0.90}{200}}} = \frac{0.02 \cdot 10\sqrt{2}}{0.3} = \frac{0.2\sqrt{2}}{0.3} =$

$= \frac{2\sqrt{2}}{3} = 0.94$



$$-z_{\frac{\alpha}{2}} = \text{qnorm}(\alpha/2) = -1.96$$

$$R = (-\infty, -1.96) \cup (1.96, \infty)$$

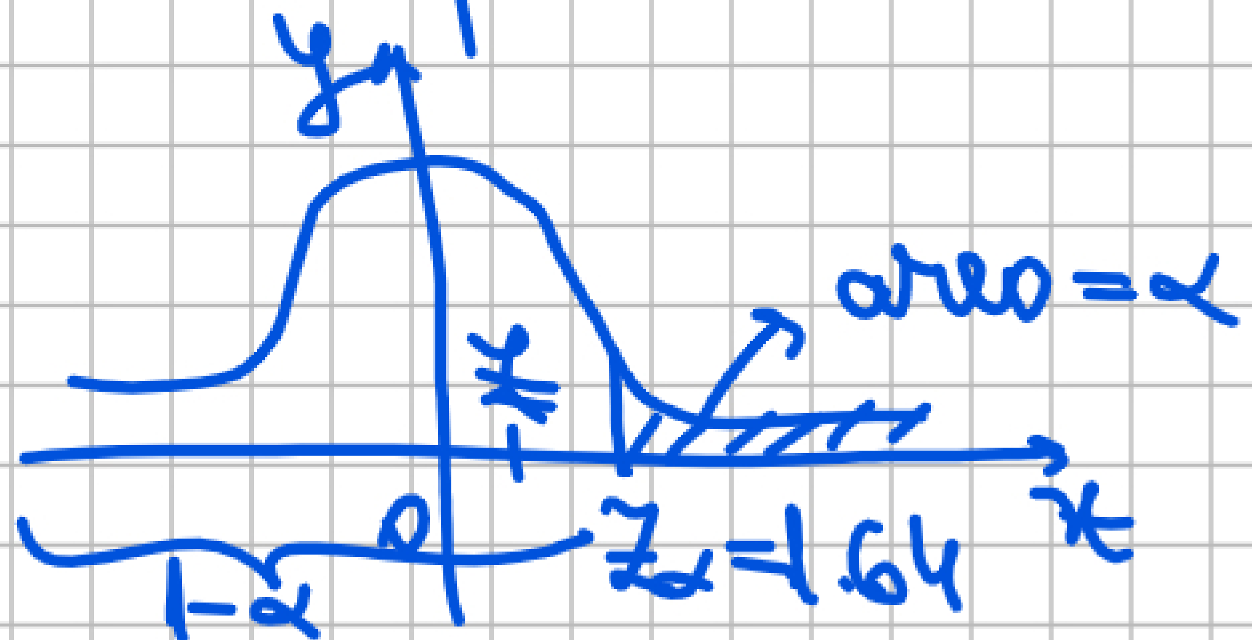
$$z = 0.94$$

$\Rightarrow z \notin R \Rightarrow$ we cannot reject H_0

$\Rightarrow$ the manufacturer's claim is true

(there isn't suff. evidence to disprove the claim)

- $H_a: p > 0.1 \Rightarrow R = (z_\alpha, \infty) = (1.64, \infty)$



$$z_\alpha = \text{qnorm}(1-\alpha) = 1.64$$

$z = 0.94 \Rightarrow z \notin R \Rightarrow H_0$ is not rejected

$\alpha = 1\% = 0.01$

$H_a: p \neq \frac{1}{10} \Rightarrow R = (-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, \infty) = (-\infty, -2.58) \cup (2.58, \infty)$

$-z_{\frac{\alpha}{2}} = \text{qnorm}(\alpha/2) = -2.58$

$z = 0.94 \Rightarrow z \notin R \Rightarrow$ we cannot reject H_0

$\alpha = 10\% \Rightarrow R = (-\infty, -1.64) \cup (1.64, \infty)$

(a) 95% confidence interval $\equiv$ acceptance region at the 5% level of significance (for the two-tailed test)

$$P\left(Z \in \left(-z_{\frac{\alpha}{2}}, z_{\frac{\alpha}{2}}\right)\right) = 1-\alpha \iff P\left(\bar{X} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

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$$95\%\ CI: \left(\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p_0(1-p_0)}{n}},\ \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right) = \left(0.3 - 1.96 \cdot \frac{0.03}{\sqrt{2}},\ 0.3 + 1.96 \cdot \frac{0.03}{\sqrt{2}}\right) =$$

$$= (0.26, 0.34)$$

$$\alpha = 1 - 95\% = 5\% \ (= 0.05)$$

$$\frac{\sigma}{\sqrt{n}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \frac{0.3}{10\sqrt{2}} = \frac{0.03}{\sqrt{2}}$$

$$(a, b) \qquad (a, \infty),\ (-\infty, b)$$

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9.11. Refer to Exercise 9.10. Having looked at the collected sample, we consider an alternative supplier. A sample of 150 items produced by the new supplier contains 13 defective items.

Is there significant evidence that the quality of items produced by the new supplier is higher than the quality of items in Exercise 9.10? What is the P-value?

2 samples: $n_1 = 200$, $\hat{p}_1 = \frac{24}{200} = \frac{3}{25} = 0.12$

$n_2 = 150$, $\hat{p}_2 = \frac{13}{150} = 0.09$

p_1, p_2 – population proportions of defective items

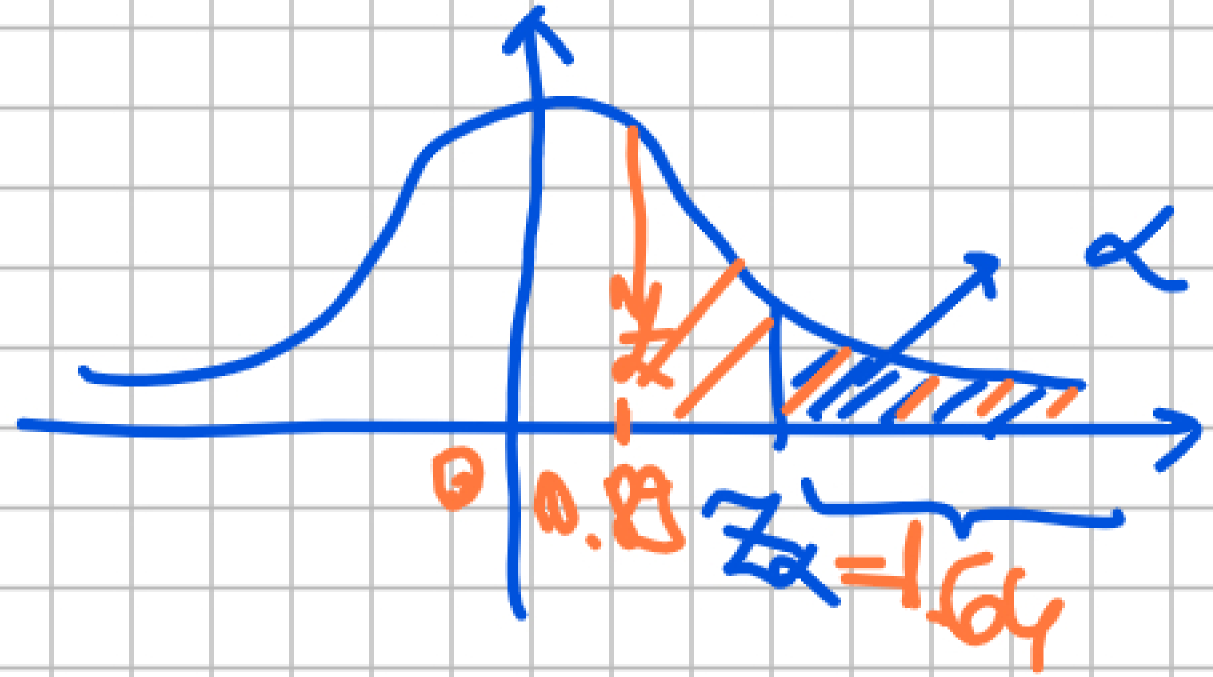
p_1 – prop. of def. items for previous supplier

p_2 – –"– of the new supplier

$H_0: p_1 = p_2$

$H_a: p_1 > p_2 \Rightarrow R = (z_\alpha, \infty)$

$\hat{p} = \frac{24+13}{200+150} = \frac{37}{350} = 0.106$



$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.12 - 0.09}{\sqrt{0.11 \cdot 0.89 \cdot \left(\frac{1}{200} + \frac{1}{150}\right)}} = 0.89$$

$\hat{p}$ – pooled sample proportion

$z_\alpha = \text{qnorm}(1-\alpha) = 1.64$

$z = 0.89, \quad R = (1.64, \infty)$

$\Rightarrow z \notin R \Rightarrow H_0$ cannot be reject (there isn't enough evidence to support the claim that the new supplier has a better quality)

p-value = area under the PDF from z to ∞

$= 1 - \text{pnorm}(z) = \text{pnorm}(z, \text{lower.tail} = F) = 0.19$

If p-value $< \alpha \Rightarrow H_0$ is rejected

p-value $> \alpha \Rightarrow H_0$ is not rejected

p-val $= 0.19 > \alpha = 0.05 \Rightarrow$

H_0 is not rejected

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9.9. Salaries of entry-level computer engineers have Normal distribution with unknown mean and variance. Three randomly selected computer engineers have salaries (in \$ 1000s):

$$30, 50, 70$$

(a) Construct a 90% confidence interval for the average salary of an entry-level computer engineer.

(b) Does this sample provide a significant evidence, at a 10% level of significance, that the average salary of all entry-level computer engineers is different from \$80,000? Explain.

(b) $n=3$ (one sample t-test)

$\alpha = 10\% = 0.1$

$H_0: \mu = 80\,000$

$H_a: \mu \neq 80\,000 \Rightarrow \mathcal{R} = (-\infty, -t_{\frac{\alpha}{2}}) \cup (t_{\frac{\alpha}{2}}, \infty)$

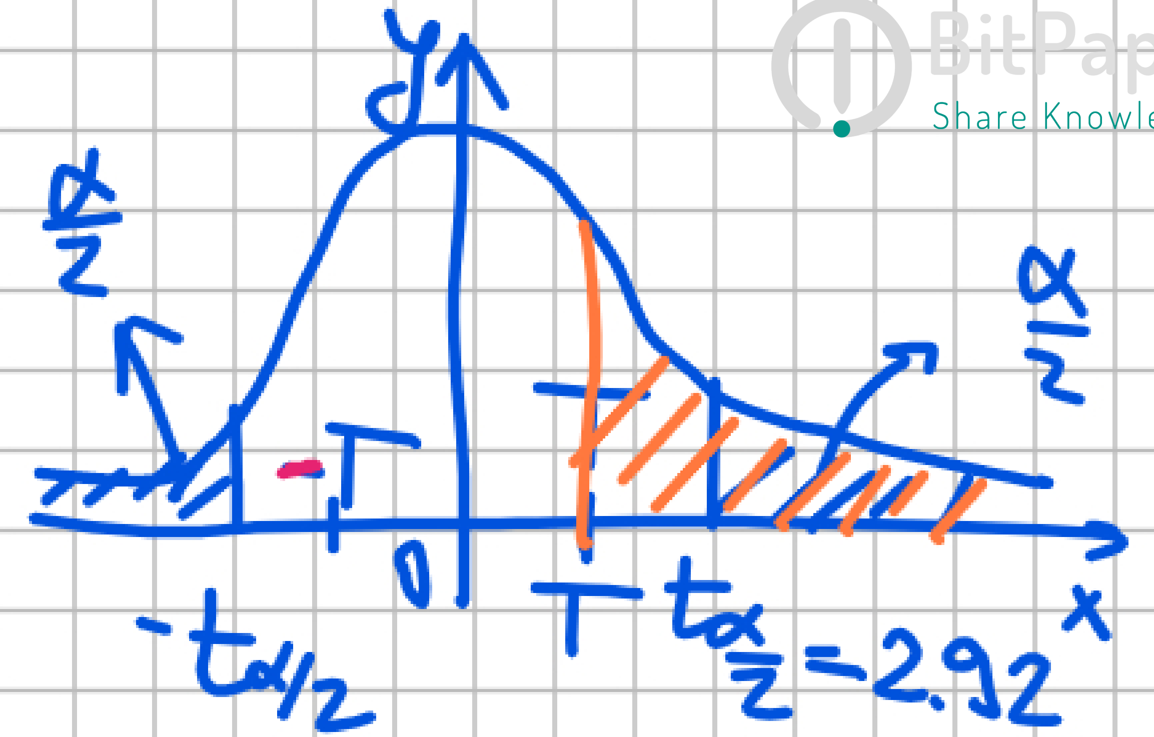
$T = \dfrac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}}$ has a Student t distribution with n-1 degrees of freedom

$\mu_0 = 80000$, S - sample std. deviation

$\overline{X}$ - sample mean

$\overline{X} = 50\,000$

$s = 20\,000$



$T = \dfrac{50\,000 - 80\,000}{\frac{20000}{\sqrt{3}}} = \dfrac{-30\,000\sqrt{3}}{20000} = -\dfrac{3\sqrt{3}}{2} = -2.60$

$-t_{\frac{\alpha}{2}} = qt(\alpha/2, n-1) = -2.92$

$\mathcal{R} = (-\infty, -2.92) \cup (2.92, \infty)$

$T = -2.60 \notin \mathcal{R} \Rightarrow$ we cannot reject H_0 (the sample doesn't provide significant evidence that the salary is different from $80000 (at the 10% level of significance))

p-value $= 2 * pt(\underset{\| }{T}^{2.60}, 2, lower.tail = F) = \underline{\underline{0.12}} > \alpha = 0.10 \Rightarrow H_0$ cannot be rejected

In R: t.test()

```
salary = c(30,50,70)
t.test(salary, mu = 80, conf.level = 0.90)
```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