LAB 1. Experiments and simulations.

In probability theory, we will be particularly interested in repeating a chance experiment a large number of times. For example, considering the experiment of rolling a die, we want to estimate the probability of obtaining a certain number. Although physically rolling the die would be a convenient way to carry out a few repetitions, it would be difficult to carry out a large number of experiments, so it is natural to turn to the computer for this task.

We must first find a computer analog of rolling a die. This is done on the computer by means of a **random number generator**. Depending upon the particular software package, the computer can be asked for a real number between 0 and 1, or an integer in a given set of consecutive integers. In the first case, the real numbers are chosen in such a way that the probability that the number lies in any particular subinterval of this unit interval is equal to the length of the subinterval. In the second case, each integer has the same probability of being chosen.

In **R**, we use the functions **runif** and **sample** for generating random numbers (see R Help on this topic).

Example: **set.seed(154)** will allow the user to generate the same values each time

runif(n,a,b) will generate n random real numbers between a and b.

sample(a:b,n,replace=T) will generate n random integers between a and b.

ASSIGNMENTS:

♣ **1.** Simulate m successive tosses of a coin and count the number of occurrences of heads. Based on your simulation results, estimated the probability of obtaining heads. Consider the following cases: $m = 10$, $m = 100$, $m = 1000$.

♣ **2.** Simulate m successive rolls of a die and count the number of occurrences of the number 6. Based on your simulation results, estimated the probability of obtaining the number 6. At first, consider a small number of repetitions ($m = 60$) and then try for a larger number of repetitions ($m = 6000$).

♣ **3.** Generate 300 random numbers in the range $[0, 100)$. How many numbers do you have in the ranges $I_1 = [0, 20)$, $I_2 = [20, 40)$, $I_3 = [40, 60)$, $I_4 = [60, 80)$ and $I_5 = [80, 100)$, respectively?

♣ **4.** Simulate the experiment of repeatedly tossing of a coin. Compute the number of tosses that are necessary to obtain two successive heads or two successive tails.

♣ **5.** In Las Vegas, a roulette wheel has 38 slots numbered 0, 00, 1, 2, ... , 36. The 0 and 00 slots are green and half of the remaining 36 slots are red and half are black. A croupier spins the wheel and throws in an ivory ball. If you bet 1 dollar on red, you win 1 dollar if the ball stops in a red slot and otherwise you lose 1 dollar. Write a program to find the total winnings for a player who makes 1000 bets on red.

♣ **6.** Consider the points $P(1, 1)$, $Q(50, 95)$ and $R(99, 1)$ in the two-dimensional plane. (1) Randomly select any point inside the triangle and consider that your current position. (2) Randomly select any one of the 3 vertex points P, Q or R. (3) Move half the distance from your current position to the selected vertex. (4) Plot the current position. (5) Repeat from step 2 (try at least 1000 repetitions).