NAME:

Grade:

| Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
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## Graph Theory

Written examination / B
27 January 2021

1. ( 0.75 p) Let $G$ be the weighted graph. Mark the edges of $G$ which form a minimum weight spanning tree of $G$, and indicate its weight.


## The total weight of the minimum

 spanning tree of $G$ is:2. (1.25p) Consider the following graph. Apply the Dijkstra's algorithm in order to compute the lightest path from $s$ to all the other nodes. Fill in the following table with the final results.

3. (1p) Which of the following trees has Prüfer sequence $5,4,3,5,1$ ?

4. (1.5p) Let $G$ be the weighted graph depicted below. Apply the Warshall algorithm to compute the matrix $W P^{[4]}$ of the lightest paths between any pair of nodes in $G$.

5. (1.5p) Find a maximum flow $f$ in the flow network $G$ with $s$ and destination $t$ depicted below. Draw $G+f$ and indicate the value $|f|$ of the maximum flow,

6. (0.5p) How many different trees with 6 nodes, there exist?
(a) 36
(b) 8
(c) 216
(d) 120
(e) 1296
7. (2p) Consider the following graph. Compute:
(I) the chromatic polynomial $c_{G}(z)$ of $G$ and
(II) what is the chromatic number of $G$ ?
(III) how many 2 -colorings has G?
(IV) how many 3 -colorings has G?


| I | $c_{G}(z)=$ |  |  |
| :--- | :--- | :--- | :--- |
| II | (a) 4 | (b) 5 | (c) 3 |
| III |  |  | (d) 6 |
| IV |  |  |  |

8. (0.5p) Let $M$ be the matching made of the edges marked in the graph $G$ depicted below:

(a) Indicate the set of $M$-saturated nodes of $G$
(b) Is $M$ a maximal? Motivate your answer.

Start: 1p

