NAME:

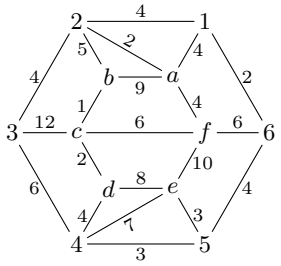
Grade:	Start	1	2	3	4	5	6	7	8

Graph Theory

Written examination / D

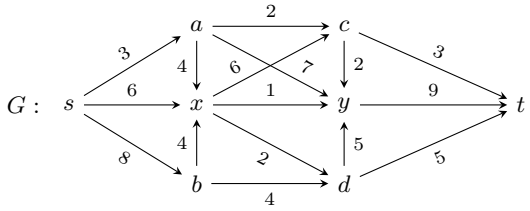
04 February 2021

1. (0.75p) Let G be the weighted graph. Mark the edges of G which form a minimum weight spanning tree of G, and indicate its weight.



The total weight of the minimum spanning tree of G is:

2. (1.25p) Consider the following graph. Apply the Dijkstra's algorithm in order to compute the lightest path from s to all the other nodes. Fill in the following table with the final results.



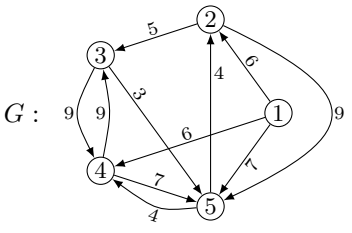
	Node							
	s	a	b	c	d	x	y	t
π								
d								

$w[s, \quad , t] =$

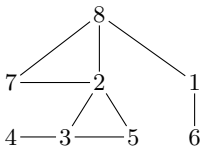
3. (0.75p) Draw the tree of which Prüfer sequence is 5,4,3,5,1?

4. (1.5p) Let G be the weighted graph depicted below. Apply the Warshall algorithm to compute the matrix $WP^{[5]}$ of the lightest paths between any pair of nodes in G.

G :

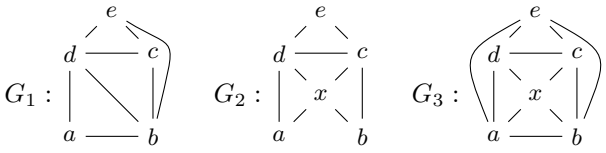


5. (2p) Consider the following graph. Compute:
(I) the chromatic polynomial $c_G(z)$ of G,
(II) what is the chromatic number of G?
(III) how many 2-colorings has G?
(IV) how many possibilities are there to color G with 3 colors?



I	$c_G(z) =$
II	(a) 4 (b) 5 (c) 3 (d) 2
III	
IV	

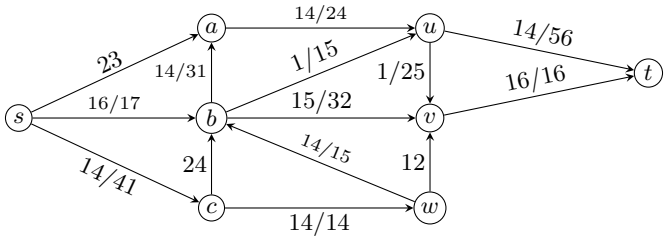
6. (0.50p) Which of the following graphs are eulerian graphs and which ones are not? Indicate a reason for each given answer. For the eulerian graphs (if any) indicate an eulerian circuit.



7. (0.50p) How many distinct trees there exist with the nodes numbered between 1 and 6?

   (a) 36 (b) 8 (c) 216 (d) 120 (e) 1296

8. (1.75p) Consider the following flow network G with flow f depicted below:



   (a) Indicate the residual network G_f.

   (b) Is f the maximum flow? If it is not, then indicate an augmenting path in G_f.

   (c) Determine a maximum flow in the flow network with source s and sink t, and indicate its value.

Start: 1p