## NAME:

Grade:

| Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
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## Graph Theory

Written examination / C
04 February 2021

1. ( 0.75 p$)$ Which is the minimum weight spanning tree of the following connected graph? (hint: apply the Kruskal algorithm)

(a) 229
(b) 216
(c) 230
(d) 234
2. (1.5p) Given the following weighted digraph. Apply the Warshall algorithm to compute the matrix $W P^{[5]}$ of the lightest paths between any two nodes of $G$.

3. ( 0.50 p ) How many different trees with 5 nodes, labeled with numbers from 1 to 5 , there exist?
(a) 10
(b) 5
(c) 25
(d) 125
(e) 3125
4. (1.25p) Consider the following graph. Apply the Dijkstra's algorithm in order to compute the lightest path from $s$ to all the other nodes. Fill in the following table with the final results.
$G$ :


|  | Node |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $s$ | $a$ | $b$ | $c$ | $x$ | $y$ | $z$ | $t$ |  |  |  |  |  |
| $\pi$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $d$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $w[s, t]=$ |  |  |  |  |  |  |  |  |  |  |  |  |  |

5. ( 0.50 p ) Is the following graph eulerian? Motivate your answer. In case it is, indicate an eulerian circuit.

6. (0.75p) Which is the Prüfer sequence of the following tree?

(a) $8,3,4,5,7,6,1,9,10,11,2$
(b) $3,4,5,7,6,1,8,2,10,11$
(c) $8,11,9,6,1,8,2,2,8$
(d) $8,11,9,6,1,8,8,2,2$
7. (2p) Consider the graph $G$. Compute:
(a) the chromatic polynomial $c_{G}(z)$ of $G$
(b) in how many ways can we color $G$ with three colors
(c) indicate the chromatic number of G,
(d) how many 4-colorings has G?

(a) $c_{G}(z)=$
(b)
(c)
(d)
8. (1.75p) Consider the following flow network $G$ with flow $f$ depicted below:

(a) Indicate the residual network $G_{f}$.
$\square$
(b) Is $f$ the maximum flow? If it is not, then indicate an augmenting path in $G_{f}$.
$\square$
(c) Determine a maximum flow in the flow network with source $s$ and $\operatorname{sink} t$, and indicate its value.

Start: 1p

