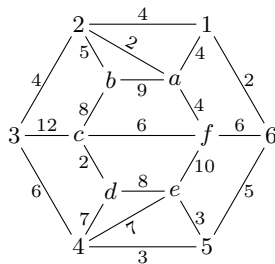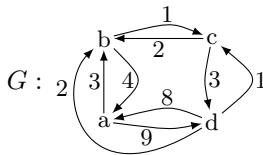# Graph Theory

Written examination / A

27 January 2021

1. (0.75p) Which is the minimum weight spanning tree of the following connected graph? (hint: apply the Kruskal algorithm)
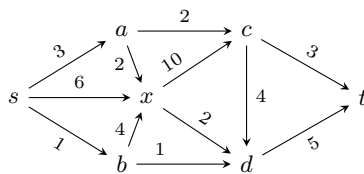

(a) 43

(b) 40

(c) 36

(d) 41

2. (1.5p) Consider the weighted graph. Apply the Warshall algorithm to compute the matrix $WP^{[4]}$ of the lightest paths between any two nodes of G.
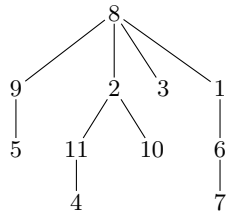

3. (1.25p) Consider the following graph. Apply the Dijkstra's algorithm in order to compute the lightest path from s to all the other nodes. Fill in the following table with the final results.

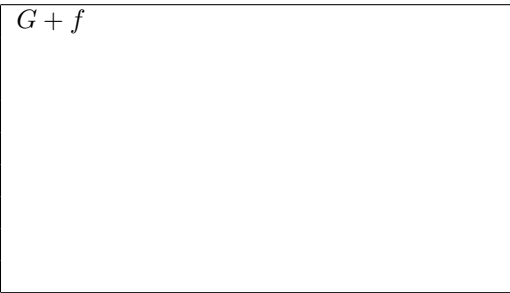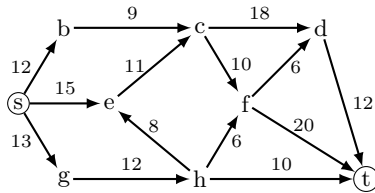|	Node						
	s	a	b	x	c	d	t
π							
d							

$$w[s, \qquad , t] =$$

4. (1p) Which is the Prüfer sequence of the following tree?

(a) $8, 3, 4, 5, 7, 6, 1, 9, 10, 11, 2$

(b) $3, 4, 5, 7, 6, 1, 8, 2, 10, 11$

(c) $8, 11, 9, 6, 1, 8, 8, 2, 2$

(d) $8, 11, 9, 6, 1, 8, 2, 2, 8$

5. (1.5p) Find a maximum flow f in the flow network G with s and destination t depicted below. Draw $G + f$ and indicate the value $|f|$ of the maximum flow,
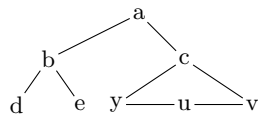

$G + f$

$|f| =$

6. (0.5p) How many different trees with 5 nodes, labeled with numbers from 1 to 5, there exist?

(a) 10   (b) 273   (c) 32   (d) 120   (e) 125
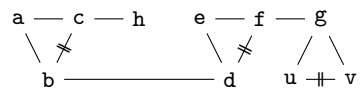
7. (2p) Consider the graph G.
   Compute:
   (a) the chromatic polynomial $c_G(z)$ of G and
   (b) in how many ways can we color G with three colors.
   (c) which is the chromatic number of G?
   (d) how many 2-colorings has G?

(a) $c_G(z) =$

(b)

(c)

(d)

2

8. (0.5p) Let M be the matching made of the edges marked in the graph G depicted below:

```
a — c — h        e — f — g          (a) Is M maximal? Motivate your answer
 \  ⧸             \  ⧸  ⁄ \
  b ──────── d     u ⊣⊢ v           (b) Indicate a maximum matching for G
```

Start: 1p