Course 6
Finite Automata/Finite State Machines

The structure and the content of the lecture is based on http://www.eecs.wsu.edu/~ananth/CptS317/Lectures/index.htm

FA with ε-Transitions

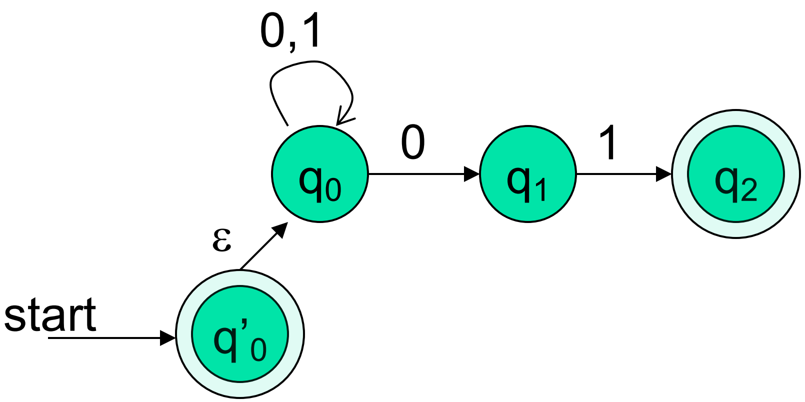
- We can allow explicit ε-transitions in finite automata
  - i.e., a transition from one state to another state without consuming any additional input symbol (then an NFA is allowed to make a transition spontaneously, without receiving an input symbol).
  - Explicit ε-transitions between different states introduce non-determinism.
  - Makes it easier sometimes to construct NFAs

Definition: ***ε-NFAs are those NFAs with at least one explicit ε-transition defined.***

- ε -NFAs have one more column in their transition table

Example of an ε-NFA

L = {w | w is empty, <u>or</u> if non-empty will end in 01}



δ_E	0	1	ε
$\rightarrow$ $*q'_0$	Ø	Ø	$\{q'_0,q_0\}$
q_0	$\{q_0,q_1\}$	$\{q_0\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$

ECLOSE(q'_0)

ECLOSE(q_0)

ECLOSE(q_1)

ECLOSE(q_2)

- ε-closure of a state q, ***ECLOSE(q)***, is the set of all states (including itself) that can be *reached* from q by repeatedly making an arbitrary number of ε-transitions (*all states reached by making an ε transition*).

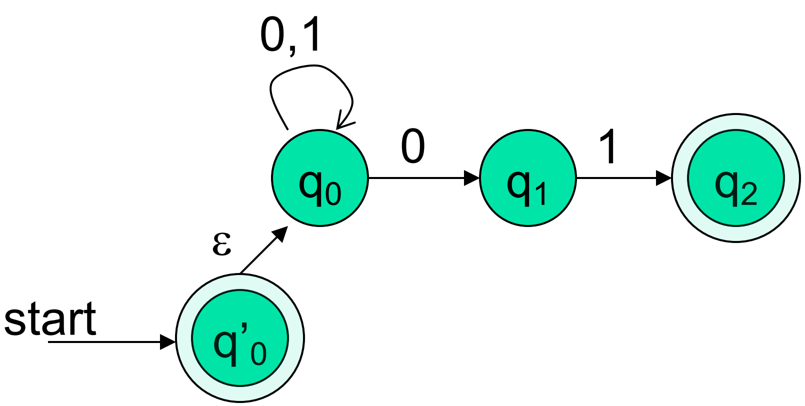
To simulate any transition:

Step 1) Go to all immediate destination states.

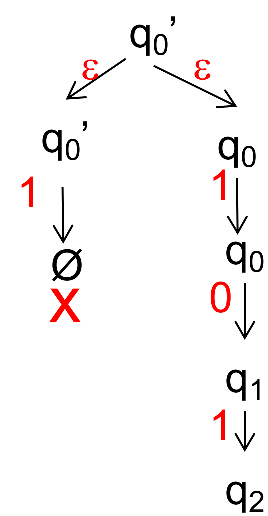
Step 2) From there go to all their ε-closure states as well.

Example of an ε-NFA

L = {w | w is empty, or if non-empty will end in 01}



Simulate for w=101:



δ_E	0	1	ε
$*q'_0$	Ø	Ø	$\{q'_0, q_0\}$
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$

ECLOSE(q'_0)

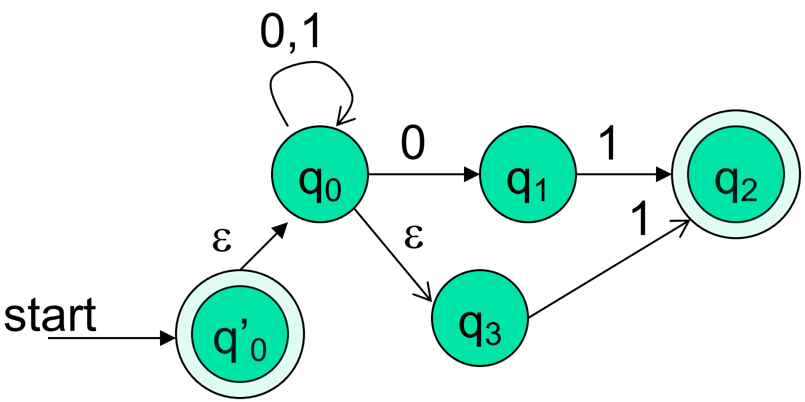
ECLOSE(q_0)

To simulate any transition:

Step 1) Go to all immediate destination states.

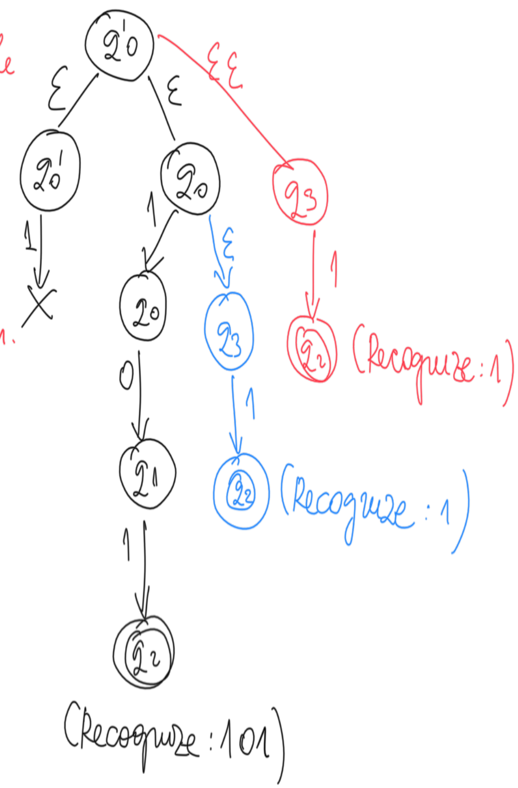
Step 2) From there go to all their ε-closure states as well.

Example of another ε-NFA



Simulate for w=101:

It is enough to construct just the path recognizing the word 101, we took the others for exemplification.



δ_E	0	1	ε
→ $*q'_0$	Ø	Ø	$\{q'_0,q_0,q_3\}$
q_0	$\{q_0,q_1\}$	$\{q_0\}$	$\{q_0,q_3\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$
q_3	Ø	$\{q_2\}$	$\{q_3\}$

Equivalency of DFA, NFA, ε-NFA

- Theorem: A language L is accepted by some ε-NFA if and only if L is accepted by some DFA ($L(DFA) = L(\varepsilon\text{−NFA})$).

- We have:
  - DFA ≡ NFA ≡ ε-NFA
  - (all accept Regular Languages)

Equivalency of DFA, NFA, ε-NFA (cont'd)

- ***Direction***: $L(DFA) \subseteq L(\varepsilon\text{−NFA})$. We turn a DFA into a ε−NFA by adding transitions $\delta(q, \varepsilon) = \emptyset$ for each $q \in Q$ (states of the DFA).
- ***Direction***: $L(DFA) \supseteq L(\varepsilon\text{−NFA})$ (see next slide).

Eliminating ε-transitions

Let $E = \{Q_E,\Sigma,\delta_E,q_0,F_E\}$ be an ε–NFA

Goal: To build DFA $D=\{Q_D,\Sigma,\delta_D,\{q_D\},F_D\}$ s.t. $L(D)=L(E)$

Construction:

1. Q_D= all reachable subsets of Q_E factoring in ε-closures
2. q_D = ECLOSE(q_0)
3. F_D=subsets S in Q_D s.t. $S \cap F_E \neq \Phi$
4. δ_D: for each subset S of Q_E and for each input symbol $a \in \Sigma$:
   - Let $R= \bigcup_{p \text{ in } s} \delta_E(p,a)$ // go to destination states
   - $\delta_D(S,a) = \bigcup_{r \text{ in } R} \text{ECLOSE}(r)$ // from there, take a union of all their ε–closures

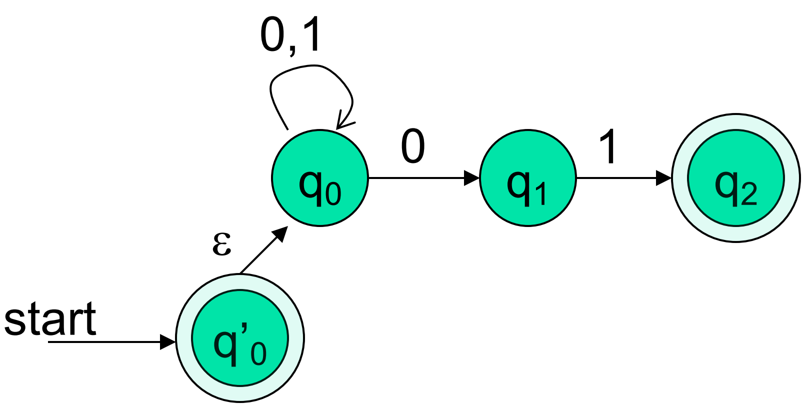
Eliminating ε-transitions (cont'd)

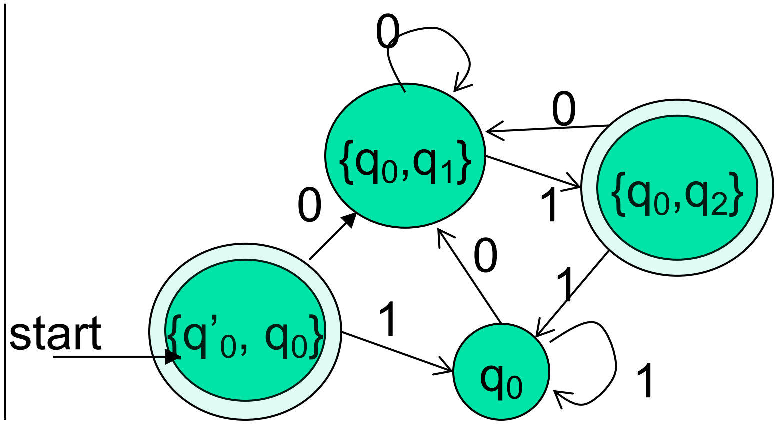
In other words:

1. Compute all ε-closures of all states of the ε-NFA
2. Compute a transition table T of the ε-NFA
3. From T compute the DFA transition table from the first state and take the resulting states as the next state in each step.

Example 1: ε-NFA → DFA

L = {w | w is empty, or if non-empty will end in 01}

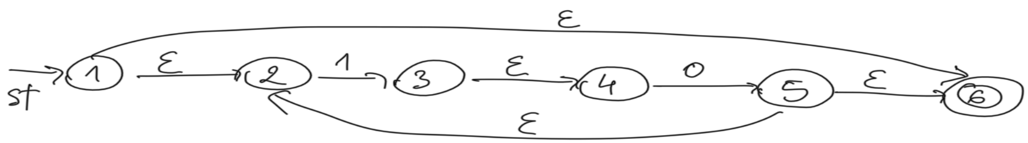




δ_E	0	1	ε
→ $*q'_0$	Ø	Ø	$\{q'_0,q_0\}$
q_0	$\{q_0,q_1\}$	$\{q_0\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$

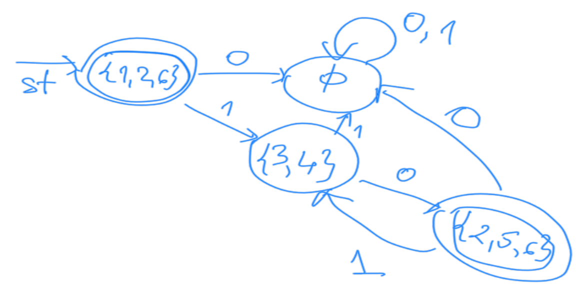
δ_D	0	1
→ $*\{q'_0,q_0\}$	$\emptyset \cup \{q_0,q_1\}$	$\emptyset \cup \{q_0\}$
$\{q_0,q_1\}$	$\{q_0,q_1\} \cup \emptyset$	$\{q_0\} \cup \{q_2\}$
$\{q_0\}$	$\{q_0,q_1\}$	$\{q_0\}$
$*\{q_0,q_2\}$	$\{q_0,q_1\} \cup \emptyset$	$\{q_0\} \cup \emptyset$

Example 2: ε-NFA → DFA



$\text{Eclose}(1) = \{1, 2, 6\}$

δ_D	0	1
$\rightarrow \{1,2,6\}$	$\phi \cup \phi \cup \phi$	$\{3,4\}$
ϕ	ϕ	ϕ
$\{3,4\}$	$\{5,2,6\} \cup$ $\{5,2,6\}$	ϕ
$*\{2,5,6\}$	$\phi \cup \phi \cup \phi$	$\{3,4\} \cup \{3,4\} \cup \phi$



Summary

- ε-NFA conversion
- Expresive power of ε-NFAs and DFAs.