Course 3
Finite Automata/Finite State Machines

The structure and the content of the lecture is based on (1) http://www.eecs.wsu.edu/~ananth/CptS317/Lectures/index.htm, (2) W. Schreiner Computability and Complexity, Lecture Notes, RISC-JKU, Austria

Excursion: Previous lecture

The Chomsky Hierarchy

We have: $\mathcal{L}_0 \supseteq \mathcal{L}_1 \supseteq \mathcal{L}_2 \supseteq \mathcal{L}_3$.

Closure properties of Chomsky families

Let $G_1 = (N_1, T_1, S_1, P_1)$, $G_2 = (N_2, T_2, S_2, P_2)$.

Closure of Chomsky families under union

The families $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ are closed under union.

Key idea in the proof

$$G_\cup = (N_1 \cup N_2 \cup S,\ T_1 \cup T_2, P_1 \cup P_2 \cup \{S \to S_1 | S_2\})$$

Closure of Chomsky families under product

The families $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ are closed under product.

Key ideas in the proof

For $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2$

$$G_p = (N_1 \cup N_2 \cup S,\ T_1 \cup T_2, P_1 \cup P_2 \cup \{S \to S_1 S_2\})$$

For $\mathcal{L}_3$

$$G_p = \left(V_{N_1 \cup N_2}, V_{T_1 \cup T_2}, S_1, P_1{}' \cup P_2\right)$$

where $P_1{}'$ is obtained from P_1 by replacing the rules $A \to p$ with $A \to pS_2$

Closure properties of Chomsky families (cont'd)

Closure of Chomsky families under Kleene closure

The families $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ are closed under Kleene closure operation.

Key ideas in the proof

For $\mathcal{L}_0, \mathcal{L}_1$

$$G^* = (V_N \cup \{S^*, X\}, V_T, S^*, P \cup \{S^* \to \lambda|S|XS, Xi \to Si|XSi, \quad i \in V_T\})$$

The new introduced rules are of type 1, so G^* does not modify the type of G.

For $\mathcal{L}_2$

$$G^* = (V_N \cup \{S^*\}, V_T, S^*, P \cup \{S^* \to S^*S|\lambda\})$$

For $\mathcal{L}_3$

$$G^* = (V_N \cup \{S^*\}, V_T, S^*, P \cup P' \cup \{S^* \to S|\lambda\})$$

where P' is obtained with category II rules, from P, namely if $A \to p \in P$ then $A \to pS \in P$.

Finite Automata

Finite Automaton (FA)

Finite state machines are everywhere!

https://www.youtube.com/watch?v=t8YKCItVDlg

Why finite automata are important?

https://www.quora.com/Why-is-it-so-important-to-have-a-good-understanding-of-automata-theory

Finite Automaton (FA)

- Informally, a state diagram that comprehensively captures all possible states and transitions that a machine can take while responding to a stream or sequence of input symbols.
- Recognizer for "Regular Languages"

- Deterministic Finite Automata (DFA)
  - The machine can exist in only one state at any given time
- Non-deterministic Finite Automata (NFA)
  - The machine can exist in multiple states at the same time

Deterministic Finite Automata - Definition

- A Deterministic Finite Automaton (DFA) consists of:
  - Q - a finite set of states
  - $\sum$ - a finite set of input symbols (alphabet)
  - q_0 - a start state (one of the elements from Q)
  - F - set of accepting states
  - $\delta : Q \times \Sigma \rightarrow Q$ – a transition function which takes a state and an input symbol as an argument and returns a state.
- A DFA is defined by the 5-tuple: $\{Q, \sum, q_0, F, \delta\}$

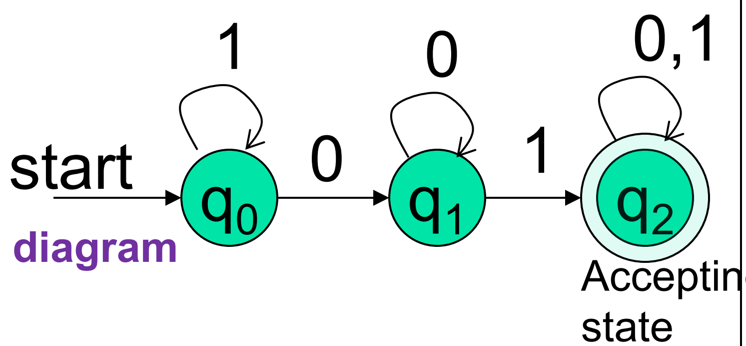
Example #1

- Build a DFA for the following language:
  - L = {w | w is a binary string that contains 01 as a substring} same as
  - L = {w | w is of the form x01y where x,y are binary strings} same as
  - L = {x01y | x,y are binary strings}
  - *Examples*: 01, 010, 011, 0011, etc.
  - *Counterexamples*: ε, 0, 1, 111000
- Steps for building a DFA to recognize L:
  - $\Sigma = \{0,1\}$
  - Decide on the non-final (non-accepting) states: Q
  - Designate start state and final (accepting) state(s): F
  - Decide on the transitions: δ

Regular expression: $(01)^*01(01)^*$

DFA for strings containing 01

- What makes this DFA deterministic?



Transition diagram

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{0,1\}$
- start state = q_0
- $F = \{q_2\}$

Transition table

symbols

	δ	0	1
Start state →	q_0	q_1	q_0
states	q_1	q_1	q_2
Accepting/final state	$^*q_2$	q_2	q_2

What if the language allows empty strings?

Example #2

- Build a DFA for the following language:
  - L = { w | w is a binary string that has exactly length 2}

See whiteboard

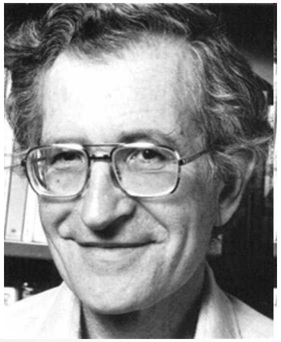
What does a DFA do on reading an input string?

- Input: a word w in Σ^*
- Question: Is w acceptable by the DFA?
- Steps:
  - Start at the "start state" q_0
  - For **every input symbol** in the sequence w do:
    - Compute the next state from the current state, given the current input symbol in w and the transition function
  - If after all symbols in w are consumed, the current state is one of the accepting states (F) then *accept w;*
  - Otherwise, *reject w.*

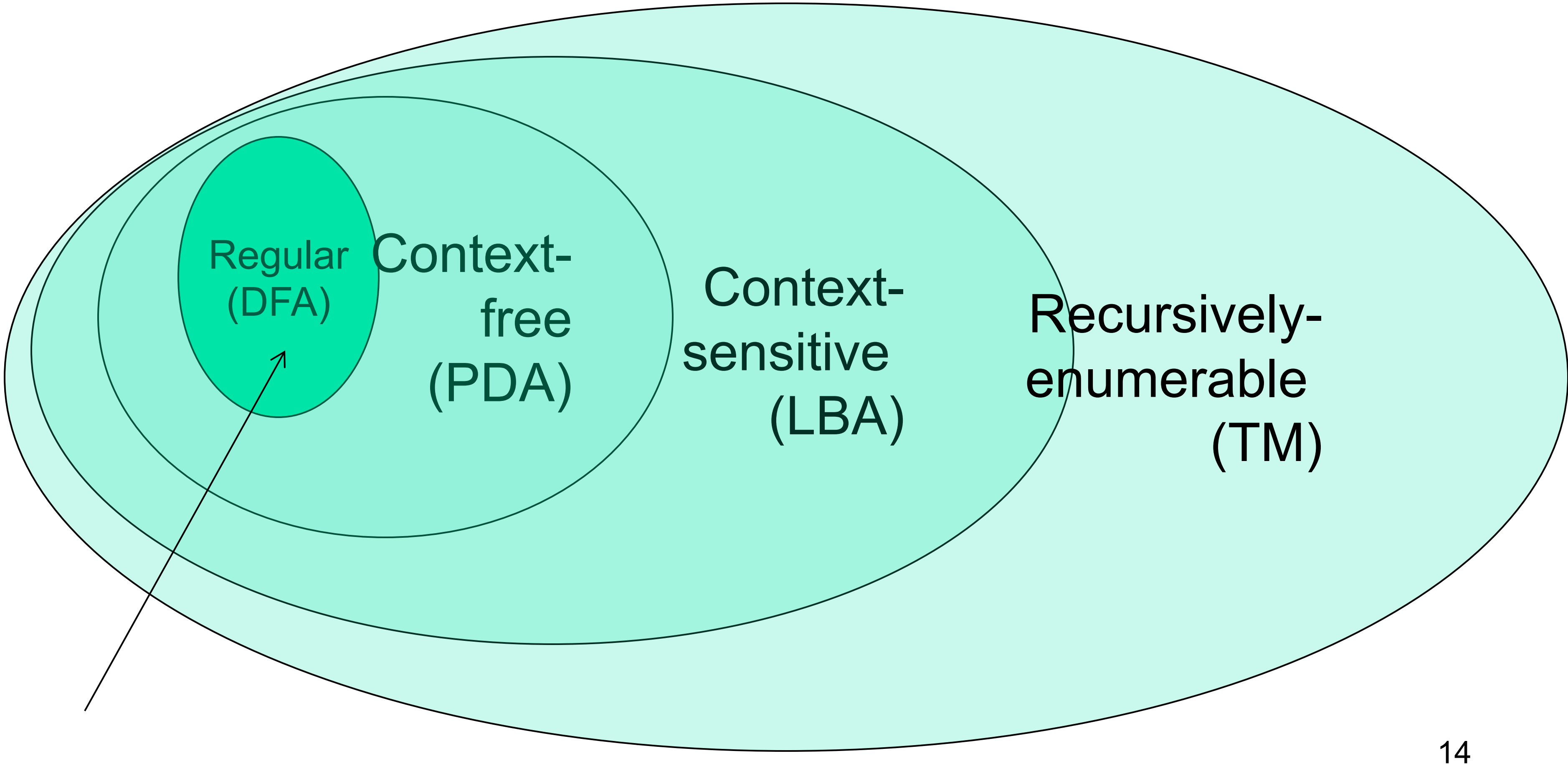
Regular Languages

- Let L(A) be a language *recognized* by a DFA A.
  - Then L(A) is called a "*Regular Language*".

The Chomsky Hierachy



Location regular languages in the Chomsky Hierarchy



Example #3: Clamping Logic

- *Problem*: A clamping circuit (https://en.wikipedia.org/wiki/Clamper_(electronics)) waits for a ”1” input, and turns on forever. However, to avoid clamping on spurious noise, we'll design a DFA that waits for *two consecutive 1s* in a row before clamping on.
- *Solution*: build a DFA for the following language:

  L = { w | w is a bit string which contains the substring 11}

  - State Design:
    - q_0 : start state (initially off), also means the most recent input was not a 1
    - q_1: has never seen 11 but the most recent input was a 1
    - q_2: has seen 11 at least once

Example #4: Even Number of Digits

- Consider the program which reads symbols from an input stream and returns true if the stream contains an even number of '0' and an even number of '1' (and no other symbol).

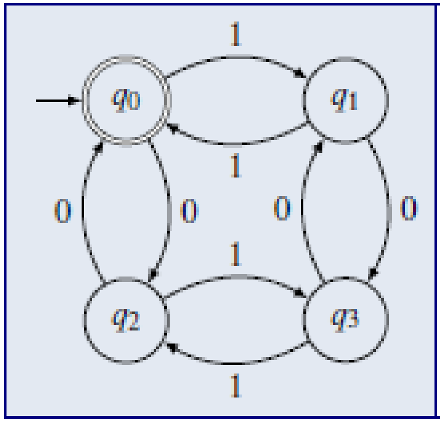
```
function EvenZerosAndOnes()
    e0, e1 ← true, true
    while input stream is not empty do
        read input
        case input of
            0: e0 ← ¬e0
            1: e1 ← ¬e1
            default: return false
        end case
    end while
    return e0 ∧ e1
end function
```

	e_0	e_1
q_0	true	true
q_1	true	false
q_2	false	true
q_2	false	false

Example #3: Even Number of Digits (cont'd)

$$M = (Q, \Sigma, \delta, q_0, F)$$
$$Q = \{q_0, q_1, q_2, q_3\}$$
$$\Sigma = \{0, 1\}$$
$$F = \{q_0\}$$

δ	0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2



Summary

- Deterministic Finite Automata