CALCULUS HANDOUT 2 - SERIES: definitions, properties, theorems

An **infinite series** is an expression of the form $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + ... + a_n + ...$
where (a_n) is a sequence of real numbers. The number a_n is called the ***n*-th term** of the series.
The ***n*-th partial sum** s_n of the series $\sum a_n$ is the sum of its first n terms: $s_n = a_1 + a_2 + \cdots + a_n$.
If (s_n) is a convergent sequence then the series $\sum a_n$ is said to be **convergent**.
If (s_n) is a divergent sequence then the series $\sum a_n$ is called **divergent**.

The **sum of an infinite series** is the limit of its sequence of finite sums: $S = \sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n$.

The series $\sum a_n$ is called **absolutely convergent** if the series $\sum |a_n|$ is convergent.
! Remark: absolute convergence implies convergence (but the converse is not true!)

♠ Geometric series:
The series $\sum_{n=0}^{\infty} ar^n$ $(a \neq 0)$ converges if and only if $|r| < 1$. In this case, its sum is $S = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.

Vanishing condition: If $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.
! Remark: If $\lim_{n\to\infty} a_n \neq 0$ or this limit does not exist, then the infinite series $\sum a_n$ is divergent.

! Remark: The converse of the vanishing condition is not true! (the *harmonic series* $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent)

Cauchy's criterion for the convergence of a series:
The series $\sum a_n$ converges if and only if for any $\varepsilon > 0$ there exists N such that for $n \geq N$ and $p \geq 1$ the following inequality holds: $|a_{n+1} + a_{n+2} + \cdots + a_{n+p}| < \varepsilon$.

Termwise addition and multiplication:
If the series $\sum a_n$ and $\sum b_n$ converge, then the series $\sum (a_n + b_n)$ and $\sum ca_n$ (with $c \in \mathbb{R}$) converge and
1. $\sum (a_n + b_n) = \sum a_n + \sum b_n$
2. $\sum ca_n = c \sum a_n$

CONVERGENCE TESTS FOR SERIES

Integral test:
Let $f : \mathbb{R}^1_+ \to \mathbb{R}^1_+$ be a decreasing function and let $a_n = f(n)$ for each $n \in \mathbb{N}$. Let $j_n = \int_1^n f(x)\,dx$. The series $\sum a_n$ converges if and only if the sequence (j_n) converges.

♠ Harmonic series: The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

Comparison test I:
Suppose that $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$. Then:
1. If $\sum b_n$ is convergent then $\sum a_n$ is convergent.
2. If $\sum a_n$ is divergent then $\sum b_n$ is divergent.

Comparison test II:
Suppose that $\sum a_n$ and $\sum b_n$ are positive-term series such that $\lim_{n\to\infty} \frac{a_n}{b_n} = L \in (0, \infty)$.
Then, $\sum a_n$ converges if and only if $\sum b_n$ converges.

Alternating series test (Leibnitz):
If (b_n) is a decreasing sequence and $\lim_{n\to\infty} b_n = 0$ then the alternating series $\sum (-1)^n \cdot b_n$ converges.

Ratio test:
Suppose that the limit $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$ exists or is infinite. Then the series $\sum a_n$
1. is absolutely convergent if $L < 1$;
2. is divergent if $L > 1$.
If $L = 1$, the ratio test is inconclusive.

Root test:
Suppose that the limit $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$ exists or is infinite. Then the series $\sum a_n$
1. is absolutely convergent if $L < 1$;
2. is divergent if $L > 1$.
If $L = 1$, the root test is inconclusive.

CALCULUS HANDOUT 2 - SERIES: examples

Ex. 1: We express the n-th partial sum of the infinite series and find the sum of the series: $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)}$.

Solution:

We can easily see that $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$. (check!)

Then

$k = 1 \Rightarrow \frac{1}{1\cdot 2} = 1 - \frac{1}{2}$

$k = 2 \Rightarrow \frac{1}{2\cdot 3} = \frac{1}{2} - \frac{1}{3}$

$k = 3 \Rightarrow \frac{1}{3\cdot 4} = \frac{1}{3} - \frac{1}{4}$

...

$k = n-1 \Rightarrow \frac{1}{(n-1)n} = \frac{1}{n-1} - \frac{1}{n}$

$k = n \Rightarrow \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

The n-th partial sum of the series is

$$\Rightarrow S_n = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}$$
$$= 1 - \frac{1}{n-1}$$

The sum of the series is

$$s = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = \lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{1}{n+1} = 1 - 0 = 1.$$

Remark: Because $s = 1 < \infty$, we have that $\sum\limits_{n=1}^{\infty}\frac{1}{n(n+1)}$ is convergent.

Ex. 2: We study the convergence of the series: a) $\sum\limits_{n=1}^{\infty}(\arcsin 1)^n$; b) $\sum\limits_{n=1}^{\infty}\frac{2n^2}{3n^2+1}$.

Solution:

a) The series $\sum\limits_{n=1}^{\infty}(\arcsin 1)^n = \sum\limits_{n=1}^{\infty}\left(\frac{\pi}{2}\right)^n$ is a geometric series with $r = \frac{\pi}{2}$.

As $|r| = \left|\frac{\pi}{2}\right| = \frac{\pi}{2} > 1$, it follows that $\sum\limits_{n=1}^{\infty}(\arcsin 1)^n$is divergent.

b) Because $\lim\limits_{n\to\infty}\frac{2n^2}{3n^2+1} = \lim\limits_{n\to\infty}\frac{2n^2}{n^2\left(3+\frac{1}{n^2}\right)} = \lim\limits_{n\to\infty}\frac{2}{3+\frac{1}{n^2}} = \frac{2}{3+0} = \frac{2}{3} \neq 0$ it results that $\sum\limits_{n=1}^{\infty}\frac{2n^2}{3n^2+1}$ is divergent.

Ex. 3: We study the convergence of the series: a) $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}$; b) $\sum\limits_{n=2}^{\infty}\frac{1}{n\ln n}$ using the integral test.

Solution:

a) Let $f : \mathbb{R}_+ \to \mathbb{R}_+$, $f(x) = \frac{1}{x^2}$. We can easily see that f is decreasing (check!) and $f(n) = \frac{1}{n^2}$. We compute

$j_n = \int\limits_1^n f(x)dx = \int\limits_1^n \frac{1}{x^2}dx = -\frac{1}{x}\Big|_1^n = -\frac{1}{n} + 1$

$\lim\limits_{n\to\infty} j_n = \lim\limits_{n\to\infty} \left(-\frac{1}{n} + 1\right) = -\lim\limits_{n\to\infty} \frac{1}{n} + \lim\limits_{n\to\infty} 1 = -0 + 1 = 1$

Because $\lim\limits_{n\to\infty} j_n = 1 < \infty$, we have that (j_n) is a convergent sequence and, by applying the integral test, we obtain that $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

b) Let $f : \mathbb{R}_+ \to \mathbb{R}_+$, $f(x) = \frac{1}{x \ln x}$. We can easily see that f is decreasing (check!) and $f(n) = \frac{1}{n \ln n}$. We compute

$j_n = \int\limits_2^n f(x)dx = \int\limits_2^n \frac{1}{x \ln x}dx = \ln(\ln x)\Big|_2^n = \ln(\ln n) - \ln(\ln 2)$

$\lim\limits_{n\to\infty} j_n = \lim\limits_{n\to\infty} (\ln(\ln n) - \ln(\ln 2)) = \lim\limits_{n\to\infty} \ln(\ln 2) - \lim\limits_{n\to\infty} \ln(\ln 2) = \infty - \ln(\ln 2) = \infty$

As $\lim\limits_{n\to\infty} j_n = \infty$, is results that (j_n) is a convergent sequence and, by applying the integral test, we obtain that $\sum\limits_{n=2}^{\infty} \frac{1}{n \ln n}$ is divergent.

Ex. 4: We study the convergence of the series: a) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+3)}}$; b) $\sum\limits_{n=1}^{\infty} 5^n \sin \frac{1}{7^n}$.

Solution:

a) Denote $a_n = \frac{1}{\sqrt{n(n+1)(n+3)}} = \frac{1}{\sqrt{n^3+4n^2+3n}}$. We can easily see that $0 < a_n = \frac{1}{\sqrt{n^3+4n^2+3n}} \le \frac{1}{\sqrt{n^3}}$.

MI: As $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} = \sum\limits_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ is a harmonic series with $p = \frac{3}{2} > 1$, it follows that $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$ is convergent and, by applying the comparison test I, we have that $\sum\limits_{n=1}^{\infty} a_n$ is convergent.

MII: We compute

$$l = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^3+4n^2+3n}}}{\frac{1}{\sqrt{n^3}}} = \lim_{n\to\infty} \frac{\sqrt{n^3}}{\sqrt{n^3+4n^2+3n}} = \lim_{n\to\infty} \frac{\sqrt{n^3}}{\sqrt{n^3\left(1+\frac{4}{n}+\frac{3}{n^2}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^3}}{\sqrt{n^3}\cdot\sqrt{1+\frac{4}{n}+\frac{3}{n^2}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{4}{n}+\frac{3}{n^2}}} = \frac{1}{\sqrt{1+0+0}} = \frac{1}{1} = 1$$

As $l = 1 \in (0, \infty)$ and $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$ is a convergent series (harmonic series with $p = \frac{3}{2} > 1$), by applying the comparison test II, it results that $\sum\limits_{n=1}^{\infty} a_n$ is also convergent.

b) Denote $a_n = 5^n \sin \frac{1}{7^n}$.

MI: Because $0 \le \sin x \le x$, $x \in \left(0, \frac{\pi}{2}\right)$ and $\frac{1}{7^n} \in \left(0, \frac{\pi}{2}\right)$, for any $n \ge 1$, we obtain

$0 \le \sin \frac{1}{7^n} \le \frac{1}{7^n} \,\Big|\, \cdot 5^n \Leftrightarrow 0 \le 5^n \sin \frac{1}{7^n} \le \frac{5^n}{7^n} \Leftrightarrow 0 \le a_n \le \left(\frac{5}{7}\right)^n$.

As $\sum\limits_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$ is a convergent series (geometric series with $|r| = \left|\frac{5}{7}\right| = \frac{5}{7} < 1$), by applying the comparison test I, we have that $\sum\limits_{n=1}^{\infty} a_n$ is convergent.

MII: Recall that $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ and we have that $\frac{1}{7^n} \to 0$. We compute

$$l = \lim_{n\to\infty} \frac{5^n \sin \frac{1}{7^n}}{\frac{5^n}{7^n}} = \lim_{n\to\infty} 5^n \cdot \frac{7^n}{5^n} \cdot \sin \frac{1}{7^n} = \lim_{n\to\infty} \frac{\sin \frac{1}{7^n}}{\frac{1}{7^n}} = 1.$$

As $l = 1 \in (0, \infty)$ and $\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$ is convergent, by applying the comparison test II, is follows that $\sum_{n=1}^{\infty} a_n$ is a convergent series.

Ex. 5: We study the convergence of the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$.

Solution:

Denote $b_n = \frac{1}{3^n}$. We can easily see that $b_n > 0$, for any $n \geq 1$ and (b_n) is a decreasing sequence, as

$$\frac{b_{n+1}}{b_n} = \frac{\frac{1}{3^{n+1}}}{\frac{1}{3^n}} = \frac{3^n}{3^{n+1}} = \frac{3^n}{3^n \cdot 3} = \frac{1}{3} < 1.$$

Furthermore, $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{3^n} = 0$.

Then, by applying Leibnitz' test for alternating series, we obtain that $\sum_{n=1}^{\infty} (-1)^n \cdot b_n$ is convergent.

Ex. 6: We study the convergence of the series: a) $\sum_{n=1}^{\infty} \frac{(n+3)!}{2^n((n+1)!)^2}$; b) $\sum_{n=1}^{\infty} (\sqrt{n^2+3n} - n)^n$.

Solution:

a) Denote $a_n = \frac{(n+3)!}{2^n((n+1)!)^2}$ and observe that $a_n > 0$, for any $n \geq 1$. We compute

$$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+4)!}{2^{n+1}((n+2)!)^2} \cdot \frac{2^n((n+1)!)^2}{(n+3)!} = \lim_{n\to\infty} \frac{(n+3)!(n+4)}{2^n \cdot 2((n+1)!)^2(n+2)^2} \cdot \frac{2^n((n+1)!)^2}{(n+3)!}$$

$$= \lim_{n\to\infty} \frac{n+4}{2(n+2)^2} = \lim_{n\to\infty} \frac{n\left(1+\frac{4}{n}\right)}{2n^2\left(1+\frac{2}{n}+\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{1+\frac{4}{n}}{2n\left(1+\frac{2}{n}+\frac{1}{n^2}\right)} = 0$$

As $l = 0 < 1$, by applying the ratio test, it follows that $\sum_{n=1}^{\infty} a_n$ is convergent.

b) Denote $a_n = (\sqrt{n^2+3n} - n)^n$ and observe that $a_n > 0$, for any $n \geq 1$. We compute

$$l = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{(\sqrt{n^2+3n}-n)^n} = \lim_{n\to\infty} (\sqrt{n^2+3n}-n) = \lim_{n\to\infty} \frac{(\sqrt{n^2+3n}-n)(\sqrt{n^2+3n}+n)}{\sqrt{n^2+3n}+n}$$

$$= \lim_{n\to\infty} \frac{(\sqrt{n^2+3n})^2 - n^2}{\sqrt{n^2+3n}+n} = \lim_{n\to\infty} \frac{n^2+3n-n^2}{\sqrt{n^2+3n}+n} = \lim_{n\to\infty} \frac{3n}{\sqrt{n^2+3n}+n} = \lim_{n\to\infty} \frac{3n}{\sqrt{n^2\left(1+\frac{3}{n}\right)}+n}$$

$$= \lim_{n\to\infty} \frac{3n}{\sqrt{n^2}\cdot\sqrt{1+\frac{3}{n}}+n} = \lim_{n\to\infty} \frac{3n}{n\sqrt{1+\frac{3}{n}}+n} = \lim_{n\to\infty} \frac{3n}{n\left(\sqrt{1+\frac{3}{n}}+1\right)} = \lim_{n\to\infty} \frac{3}{\sqrt{1+\frac{3}{n}}+1}$$

$$= \frac{3}{\sqrt{1+0}+1} = \frac{3}{2}$$

As $l = \frac{3}{2} > 1$, by applying the root test, we have that $\sum_{n=1}^{\infty} a_n$ is divergent.

CALCULUS HANDOUT 2 - SERIES: exercises

1. Express the n-th partial sum of the infinite series and find the sum of the series.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n^2+n}}$ 3. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2-1}$ 5. $\displaystyle\sum_{n=1}^{\infty}\ln\frac{n+1}{n}$ 7. $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n(n+1)(n+2)}$ 9. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{9n^2+3n-2}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$ 4. $\displaystyle\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$ 6. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$ 8. $\displaystyle\sum_{n=1}^{\infty}\frac{6n}{n^4-5n^2+4}$ 10. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{16n^2-8n-3}$

2. Determine wether the following series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sin\frac{1}{n}}$ 3. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{3}{e}\right)^n$ 5. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n}-\frac{1}{2^n}\right)$ 7. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n+3^n}$ 9. $\displaystyle\sum_{n=1}^{\infty}(\arctan 1)^n$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{n+1}$ 4. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}}{\ln(n+1)}$ 6. $\displaystyle\sum_{n=1}^{\infty}\frac{1+2^n+5^n}{3^n}$ 8. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\ln n}$ 10. $\displaystyle\sum_{n=1}^{\infty}\left[\left(\frac{7}{11}\right)^n-\left(\frac{3}{5}\right)^n\right]$

3. Use the integral test to test the following series for convergence.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{e^n}$ 2. $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$ 3. $\displaystyle\sum_{n=1}^{\infty}\frac{\arctan n}{n^2+1}$ 4. $\displaystyle\sum_{n=1}^{\infty}\frac{2^{1/n}}{n^2}$ 5. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(\ln n)^p}$, $p\in\mathbb{R}$

4. Use comparison tests to determine wether the following series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+n+1}$ 5. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+1}}$ 9. $\displaystyle\sum_{n=1}^{\infty}\frac{\sin^2 n}{n^2+1}$ 13. $\displaystyle\sum_{n=1}^{\infty}\frac{e^{1/n}}{n}$ 17. $\displaystyle\sum_{n=1}^{\infty}3^n\sin\frac{\pi}{5^n}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{n^3+1}{n^4+2}$ 6. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+n}$ 10. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos^2 n}{3^n}$ 14. $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$ 18. $\displaystyle\sum_{n=1}^{\infty}\frac{(n+1)^n}{n^{n+1}}$

3. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+n^{3/2}}$ 7. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\ln n}$ 11. $\displaystyle\sum_{n=1}^{\infty}\frac{n+2^n}{n+3^n}$ 15. $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2-1}{n^2\cdot 3^n}$ 19. $\displaystyle\sum_{n=1}^{\infty}\arctan\frac{1}{n^2+n+1}$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{10n^2}{n^4+1}$ 8. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n-\ln n}$ 12. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n+3^n}$ 16. $\displaystyle\sum_{n=1}^{\infty}\frac{2+\sin n}{n^2}$ 20. $\displaystyle\sum_{n=1}^{\infty}\ln\left(1+\frac{3}{n^2+4n}\right)$

5. Determine wether or not the following alternating series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ 3. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{\sqrt{n^2+2}}$ 5. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{2^n}$ 7. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\sin\frac{1}{n}$ 9. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n!}{(2n)!}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{3n^2+2}$ 4. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{\sqrt{n}}$ 6. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[n]{2}}$ 8. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[n]{n}}$ 10. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^{3/2}}$

6. Using the root test or the ratio test, determine wether the following series are convergent or divergent.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{n^n}$ 3. $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$ 5. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^n$ 7. $\displaystyle\sum_{n=1}^{\infty}\frac{a^n}{n^2}$, $a\in\mathbb{R}$ 9. $\displaystyle\sum_{n=1}^{\infty}a^n\left(1+\frac{1}{n}\right)^n$, $a>0$

2. $\displaystyle\sum_{n=1}^{\infty}3^{-\sqrt{n^2-2}}$ 4. $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2 n^2}{(2n)!}$ 6. $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{n!n}$ 8. $\displaystyle\sum_{n=1}^{\infty}\frac{(an)^n}{n!}$, $a\in\mathbb{R}$ 10. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{an+1}{bn+2}\right)^n$, $a,b>0$

Extra exercise

7. Determine wether the following series are absolutely convergent, simply convergent or divergent.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$, $p\in\mathbb{R}$ 3. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n(n+1)}}$ 5. $\displaystyle\sum_{n=1}^{\infty}\frac{(-10)^n}{n!}$ 7. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^n}$ 9. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n^n}{3^{n^2}}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{n}$ 4. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt[n]{n}}$ 6. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sin n}{n}$ 8. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt[n]{n}}$ 10. $\displaystyle\sum_{n=1}^{\infty}\frac{(n+2)!}{3^n(n!)^2}$